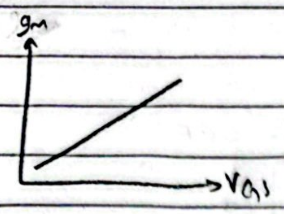
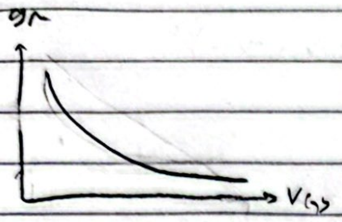
1) $g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) \cdot I_D} = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{th}) = \frac{2 I_D}{V_{GS} - V_{th}}$

a)



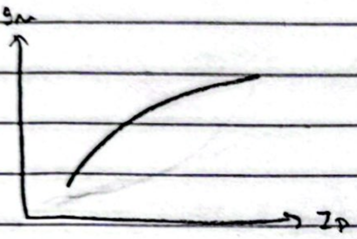
(linear)

b)



(inversely sqrt), if V_{GS} increases by a factor of 4, then g_m will decrease by 2. (halved)

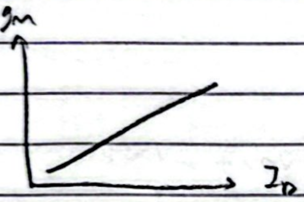
c)



sqrt

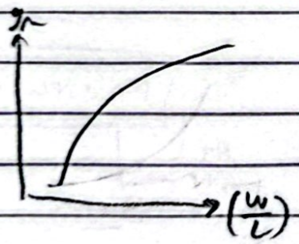
If I_D increases by a factor of 4, then g_m will increase by 2.

d)



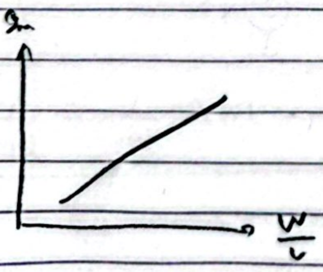
(linear)

e)



(sqrt) (same as part c)

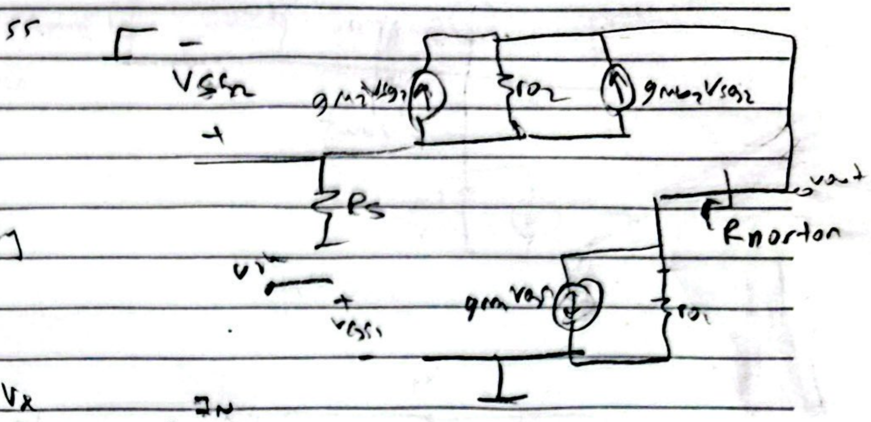
f)



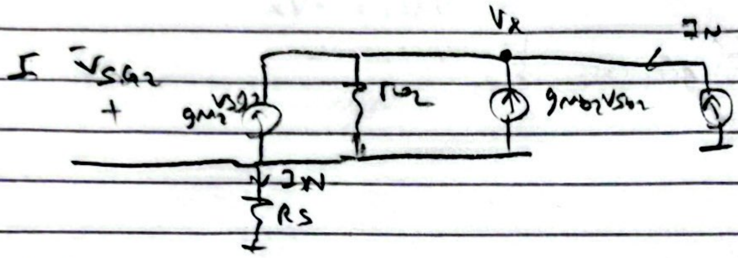
linear

2) Norton eq. ???

a) couldn't figured out how to show M1 as a Norton eq. circuit. I solve by a different approach



M1 don't suffer from body effect



$V_{S2} = I_N R_S$

$V_{SG2} = I_N R_S$

$V_{SB2} = V_{S2}$

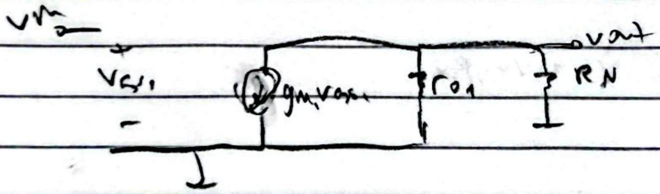
at V_X:

$$-I_N + \frac{V_X - V_{S2}}{r_{o2}} - g_{m2}\overbrace{V_{SG2}}^{I_N R_S} - g_{mb2}\overbrace{V_{SB2}}^{I_N R_S} = 0$$

$$I_N\left[1 + R_S\left(\frac{1}{r_{o2}} + g_{m2} + g_{mb2}\right)\right] = \frac{V_X}{r_{o2}}$$

$$\frac{V_X}{I_N} = r_{o2}\left[1 + R_S\left(\frac{1}{r_{o2}} + g_{m2} + g_{mb2}\right)\right] = R_N$$

v_{in}



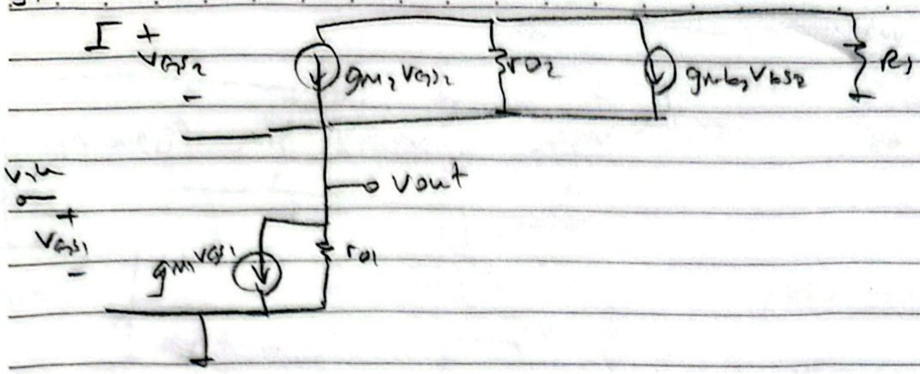
$V_{GS1} = V_{in}$

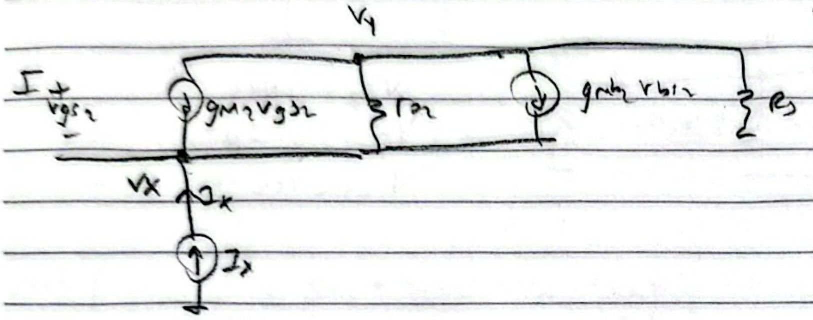
at v_{out}:

$$\frac{v_{out}}{R_N} + \frac{v_{out}}{r_{o1}} + g_{m1} v_{in} = 0 \rightarrow v_{out}\left(\frac{1}{R_N} + \frac{1}{r_{o2}}\right) = -g_{m1} v_{in}$$

$$\boxed{\frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{\left(\frac{1}{R_N} + \frac{1}{r_{o2}}\right)}}$$

b)





$V_{gs2} = -V_x$

$V_{bs2} = -V_x$

at V_x:

$$-I_x + g_{m2}V_x + g_{mb2}V_x + \frac{V_x - V_y}{r_{o2}} = 0$$

at V_y:

$$\frac{V_y}{R_J} - g_{m2}V_x - g_{mb2}V_x + \frac{V_y - V_x}{r_{o2}} = 0$$

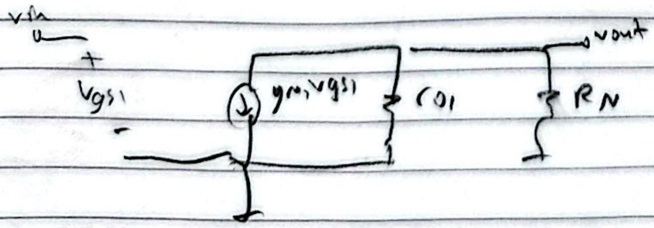
$$V_y\left(\frac{1}{R_J} + \frac{1}{r_{o2}}\right) = V_x\left(\frac{1}{r_{o2}} + g_{m2} + g_{mb2}\right)$$

$$V_y = V_x \cdot \underbrace{\frac{\frac{1}{r_{o2}} + g_{m2} + g_{mb2}}{\frac{1}{R_J} + \frac{1}{r_{o2}}}}_{\text{say } R_x} \Rightarrow V_y = V_x \cdot R_x$$

$$\Rightarrow V_x\left(g_{m2} + g_{mb2} + \frac{1}{r_{o2}} - \frac{R_x}{r_{o2}}\right) = I_x$$

$$\boxed{R_N = \frac{V_x}{I_x} = \frac{1}{g_{m2} + g_{mb2} + \frac{1}{r_{o2}} - \frac{R_x}{r_{o2}}}}$$

$V_{gs1} = V_{in}$

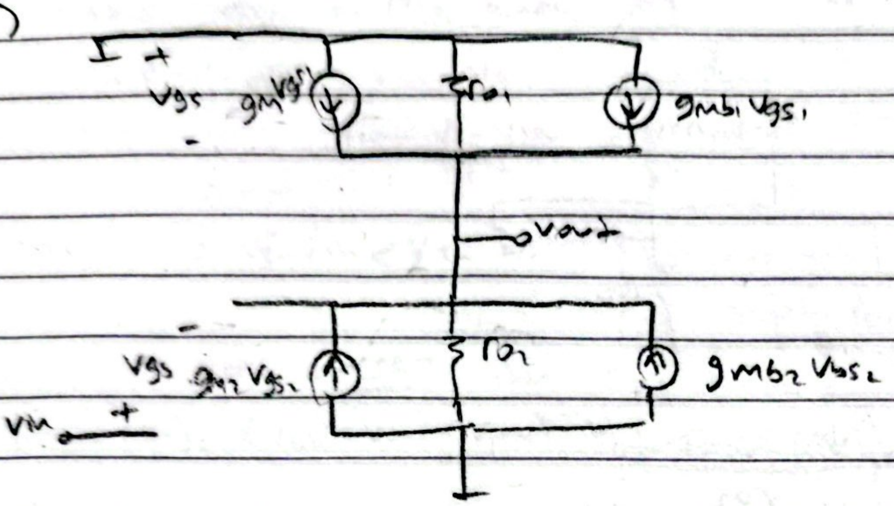


at V_{out}: $\frac{V_{out}}{R_N} + \frac{V_{out}}{r_{o1}} + g_{m1}V_{in} = 0$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{-g_{m1}}{\frac{1}{R_N} + \frac{1}{r_{o1}}}}$$

3)

c)



$V_{gs_1} = -V_{out} = V_{bs_1}$

$V_{gs_2} = V_{in} - V_{out}$

$V_{bs_2} = V_{DD} - V_{out}$

<u>V_{out}:</u>

$$\frac{v_{out}}{r_{o_1}} + \frac{v_{out}}{r_{o_2}} + v_{out}(g_{m_2} + g_{mb_2}) + (v_{out} - v_{in})g_{m_1} + (v_{out} - V_{DD})\, g_{mb_1}$$

$$v_{out}\left(\frac{1}{r_{o_1}} + \frac{1}{r_{o_2}} + g_{m_2} + g_{mb_2} + g_{m_1} + g_{mb_1}\right) = v_{in} g_{m_1} + V_{DD}\, g_{mb_1}$$

$$\frac{v_{out}}{v_{in}} = \frac{1}{v_{in}} \cdot \left(\frac{v_{in} g_{m_1} + V_{DD}\, g_{mb_1}}{\frac{1}{r_{o_1}} + \frac{1}{r_{o_2}} + g_{m_1} + g_{m_2} + g_{mb_1} + g_{mb_2}}\right)$$

why is there an offset term in the gain equation?

4) $V_{D2} = V_{G2} = V_{DD} \Rightarrow M_2$ always SAT

For M_1: $V_{D1} = V_{out}$, $V_{G1} = V_B$, $V_{S1} = V_{in}$

$V_{DS} = V_{out} - V_{in}$ $\quad V_{GS} = V_B - V_{in}$

Assume OFF state

$V_{GS} < V_{th} \Leftrightarrow V_B - V_{in} < V_{th} \Rightarrow \boxed{V_{in} > V_B - V_{th}}$ M_1 is OFF

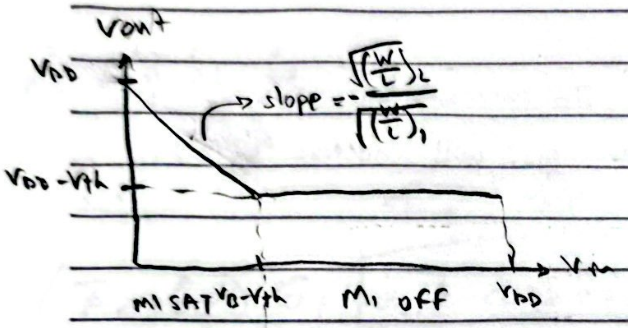
$\boxed{\text{so } V_{out} = V_{DD} - V_{th}}$

now assume M_1 is SAT, then $I_{D2} = I_{D1}$

$$\cancel{\tfrac{1}{2}} \cancel{\mu_n C_{ox}} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{th})^2 = \cancel{\tfrac{1}{2}} \cancel{\mu_n C_{ox}} \left(\frac{W}{L}\right)_1 (V_B - V_{in} - V_{th})^2$$

$$\Rightarrow \sqrt{\left(\frac{W}{L}\right)_2} \cdot (V_{DD} - V_{out} - V_{th}) = \sqrt{\left(\frac{W}{L}\right)_1} (V_B - V_{in} - V_{th})$$

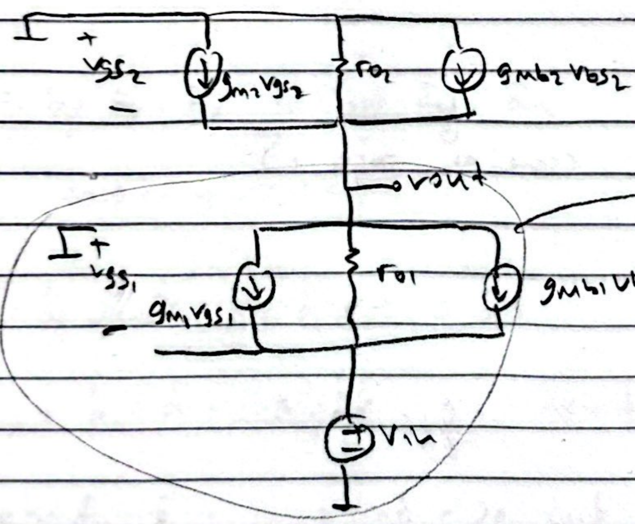
$$\sqrt{\left(\frac{W}{L}\right)_2}\, V_{out} = \sqrt{\left(\frac{W}{L}\right)_1} (V_{in} - V_B) + V_{th}\left(\sqrt{\left(\frac{W}{L}\right)_1} - \sqrt{\left(\frac{W}{L}\right)_2}\right) + V_{DD}\sqrt{\left(\frac{W}{L}\right)_2}$$



M1 SAT | M1 OFF
M2 SAT | M2 SAT

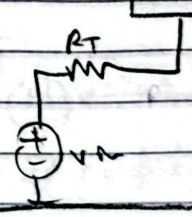
$V_{gs2} = -V_{out} = V_{bs2}$

$V_{gs1} = -V_{in} = V_{bs1}$



also

M2 circuit



$\boxed{R_T = r_{o1} \parallel \frac{1}{g_{m1}} \parallel \frac{1}{g_{mb1}}}$

KCL at Vout:

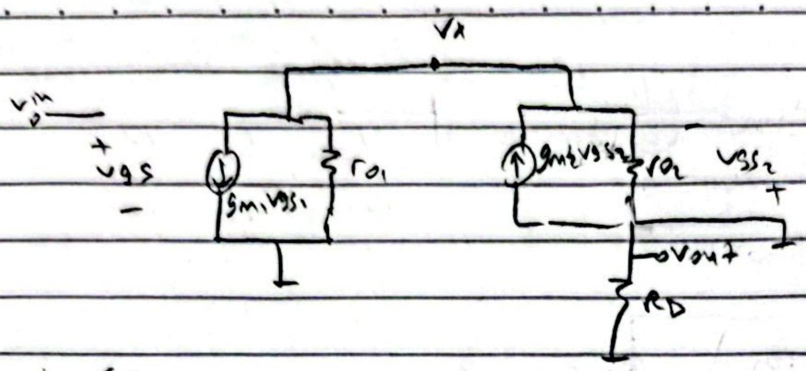
$$\frac{v_{out}}{r_{o2}} + v_{out}(g_{m2} + g_{mb2}) + \frac{v_{out} - v_{in}}{r_{o1}} - v_{in}(g_{m1} + g_{mb1}) = 0$$

$$= v_{out}\left(\frac{1}{r_{o2}} + g_{m2} + g_{mb2} + \frac{1}{r_{o1}}\right) = v_{in}\left(\frac{1}{r_{o1}} + g_{m1} + g_{mb1}\right)$$

$$\boxed{\frac{v_{out}}{v_{in}} = \frac{\frac{1}{r_{o1}} + g_{m1} + g_{mb1}}{\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{m2} + g_{mb2}}}$$

5)

b)



$V_{gs2} = -V_x$

$V_{gs1} = V_{in}$

KCL at V_x:

$$\frac{V_x}{r_{o1}} + g_{m1}V_{in} + g_{m2}V_x + \frac{V_x - V_{out}}{r_{o2}} = 0$$

$$V_x\left(\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + g_{m2}\right) + g_{m1}V_{in} - \frac{V_{out}}{r_{o2}} = 0$$

KCL at V_{out}:

$$\frac{V_{out}}{R_D} + \frac{V_{out} - V_x}{r_{o2}} - g_{m2}V_x = 0 \Rightarrow \boxed{V_x = V_{out}\left(\frac{\frac{1}{R_D} + \frac{1}{r_{o2}}}{g_{m2} + \frac{1}{r_{o2}}}\right)}$$

$$\Rightarrow V_{out}\left[\left(\frac{1}{r_{o2}} + g_{m2} + \frac{1}{r_{o1}}\right)\cdot\left(\frac{\frac{1}{R_D} + \frac{1}{r_{o2}}}{g_{m2} + \frac{1}{r_{o2}}}\right) - \frac{1}{r_{o2}}\right] = -g_{m1}V_{in}$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{-g_{m1}}{\left[\left(\frac{1}{r_{o2}} + g_{m2} + \frac{1}{r_{o1}}\right)\cdot\left(\frac{\frac{1}{R_D} + \frac{1}{r_{o2}}}{g_{m2} + \frac{1}{r_{o2}}}\right) - \frac{1}{r_{o2}}\right]}}$$

a)