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Department: EEE

Course Code: EEE321

Section: 02

Experiment Number: 04

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Lab 4 Report

Introduction:

The purpose of this lab is to analyze Fourier series coefficients of continuous time signals and observing the effect of operations on Fourier series coefficients.

Analysis:

Part 1.1 Implementing Fourier Series Analysis

In this part, a function is written in MATLAB in order to calculate the Fourier series coefficients of the inputted continuous-time signal. Function basically takes two inputs which are input signal and number of two sided Fourier series coefficients. Codes for this and following parts are given in the Appendix.

Part 1.2 Testing the Function

The Fourier series coefficients' derivations for $x_1(t)$ and $x_2(t)$ are given at the end of this report. Fig. 1 shows the Fourier series coefficients of $x_1(t)$ signal and Fig. 2 shows the Fourier series coefficients of $x_2(t)$ signal.

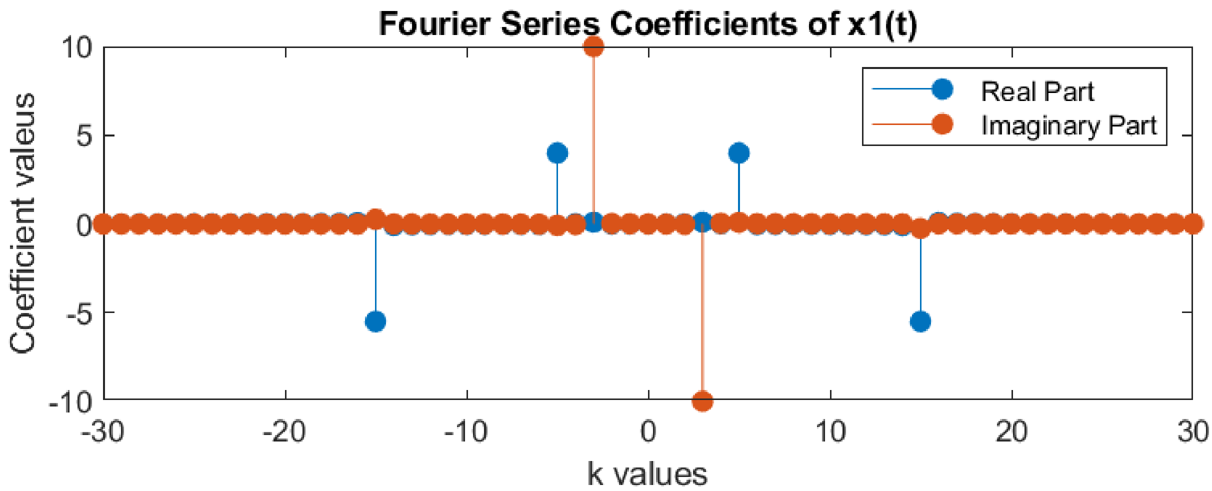

Fig. 1: Real and Imaginary Fourier series coefficients of $x_1(t)$

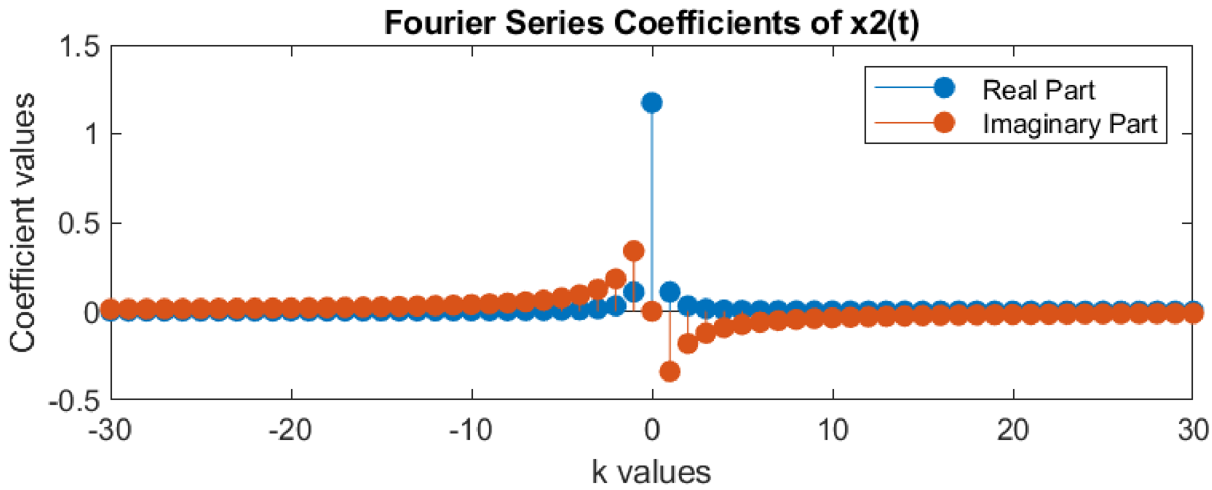

Fig. 2: Real and Imaginary Fourier series coefficients of $x_2(t)$

Derived Fourier series coefficients of $x_1(t)$ signal are given below.

$$a_{\pm 3} = \frac{10}{j}$$

$$a_{\pm 5} = 4$$

$$a_{\pm 15} = \frac{-11}{2}$$

Also, fundamental period of $x_1(t)$ signal is found as 2 seconds.

Derived Fourier series coefficients of $x_2(t)$ signal are given below.

$$a_0 = 1.1752$$

$$a_1 = 0.1 - j0.32$$

$$a_{-1} = 0.1 + j0.32$$

Comparing the derived results with the ones on the graph for $x_1(t)$ and $x_2(t)$, derived and graph results are same with each other with a slight difference.

For a signal to hold Parseval's Relation, it's energy in time domain and frequency domain has to be same with each other. Looking at the output of the code, both of the energies in time and frequency domain are equal to each other with a value of 292.

- Time-domain energy of x1: 292.509
- Frequency-domain energy of x1: 292.2164

Part 2

In this part, Fourier series coefficients of $x_3(t)$ signal and 5 other signals written in terms of $x_3(t)$ signal are going to be analyzed. Below given the $x_3(t)$ signal.

$$x_3(t) = r(t) - r(t-3) - 3u(t-3) \text{periodic with } T = 4 \text{ s}$$

Fig. 3 shows the Fourier series coefficients of the $x_3(t)$ signal.

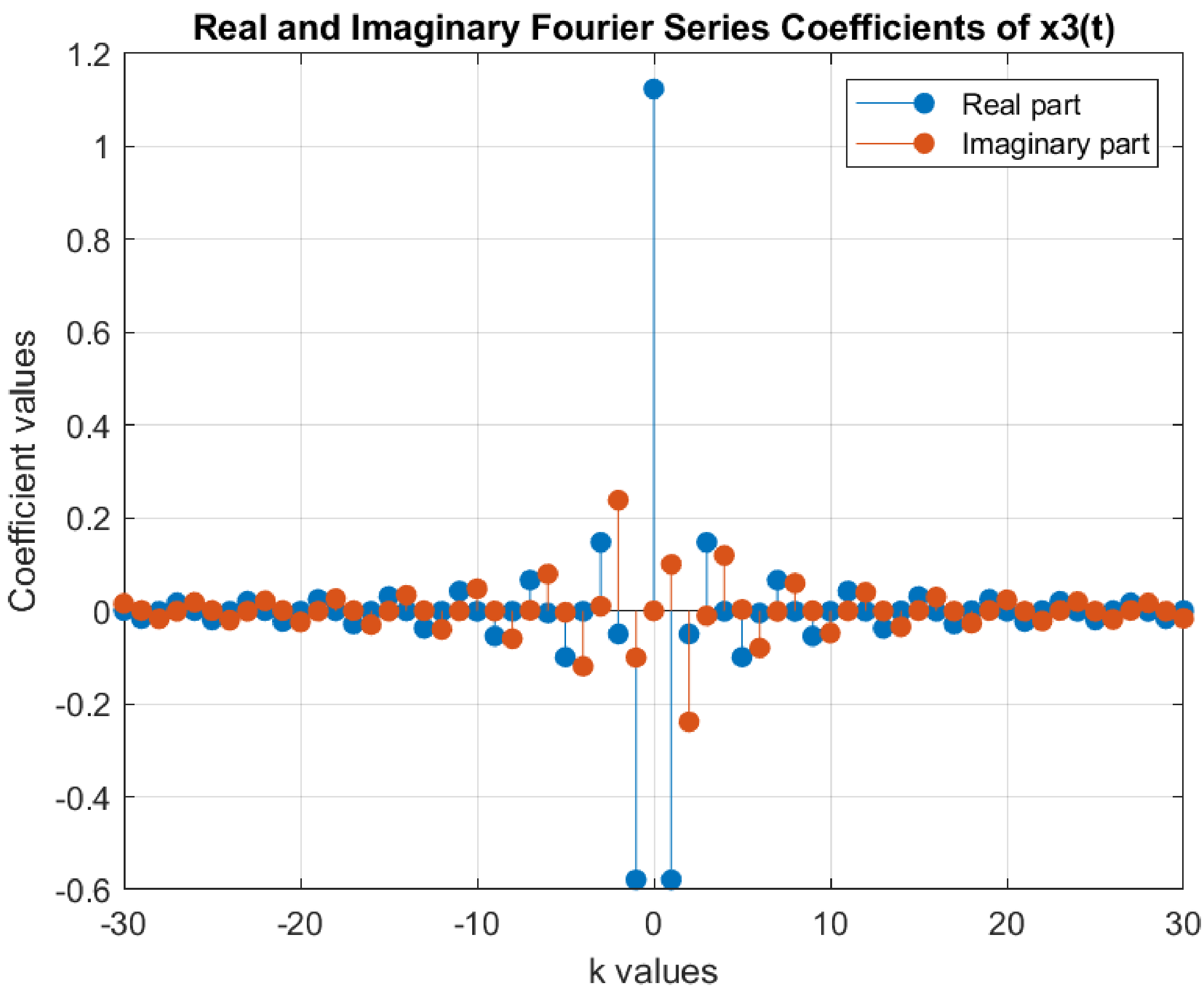

Fig. 3: Fourier series coefficients of the $x_3(t)$ signal

Fig. 4 shows the Fourier series coefficients of the $z_1(t) = x_3(-t)$ signal.

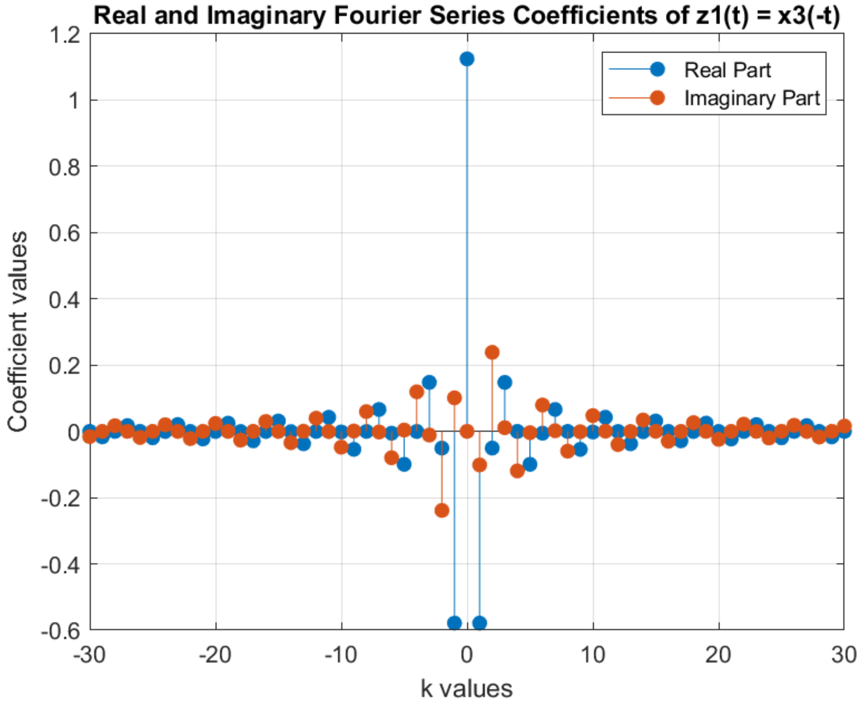

Fig. 4: Fourier series coefficients of the $z_1(t)$ signal

Comparing $x_3(t)$ and $z_1(t)$ graphs, it is seen that also Fourier series coefficients are reversed with respect to y-axis.

Fig. 5 shows the Fourier series coefficients of the $z_2(t) = \frac{dx_3(t)}{dt}$ signal.

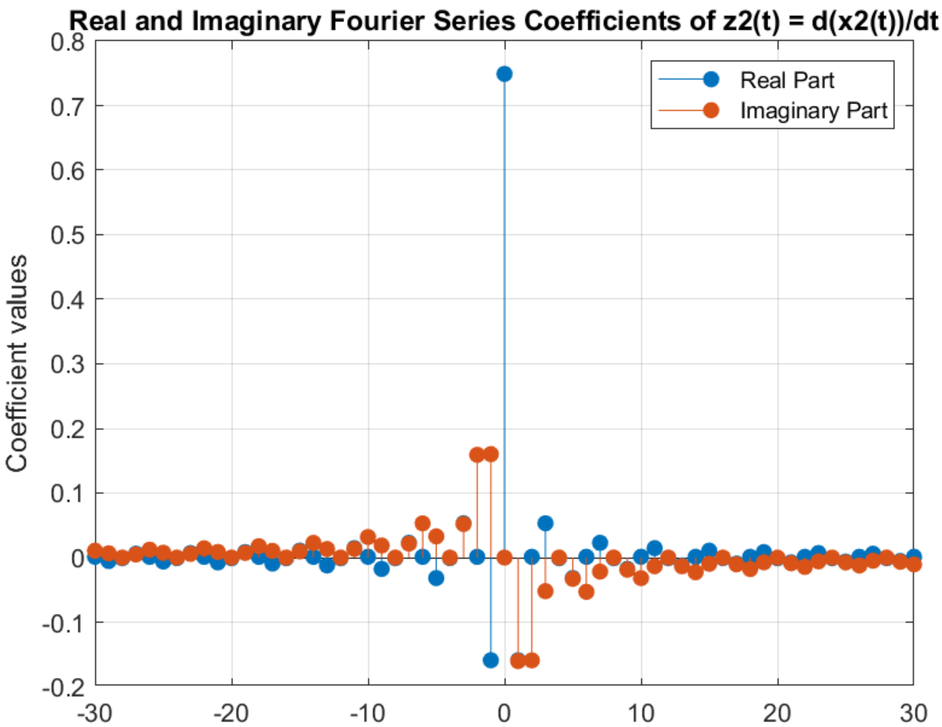

Fig. 5: Fourier series coefficients of the $z_2(t)$ signal

Comparing $z_2(t)$ graph to $x_3(t)$, in general they are same, only around the origin point, there is a change in the real and imaginary part Fourier series coefficient values.

Fig. 6 shows the Fourier series coefficients of the $z_3(t) = x_3(t+2)$ signal.

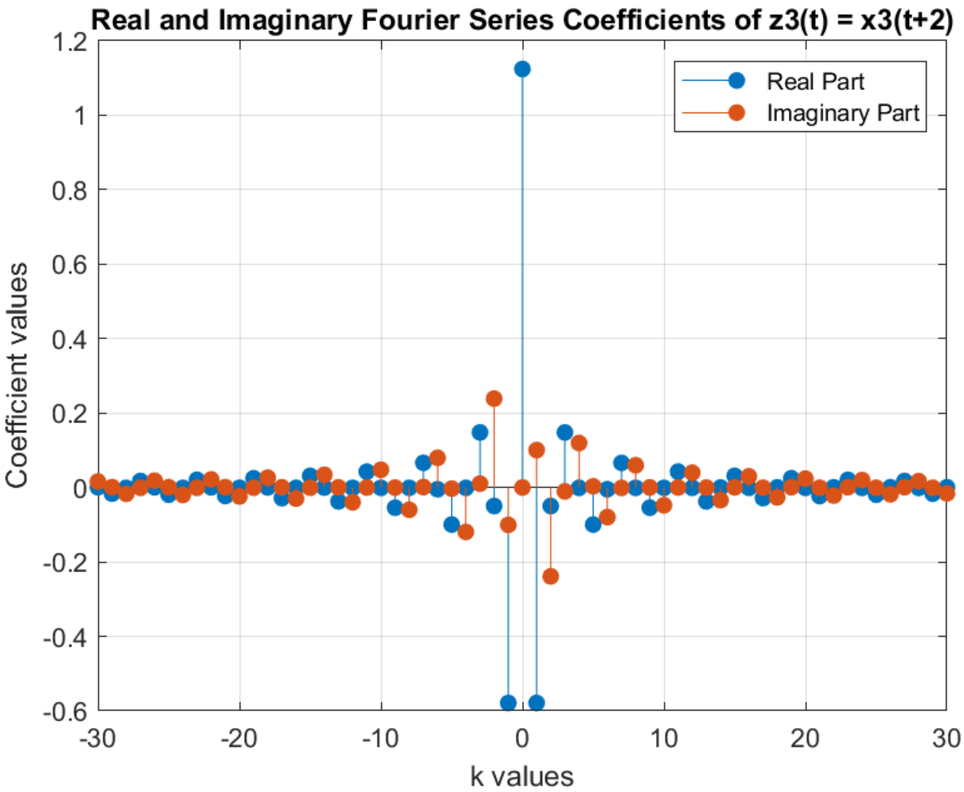

Fig. 6: Fourier series coefficients of the $z_3(t)$ signal

Comparing $z_3(t)$ graph to $x_3(t)$, it can be said that time shift in a continuous-time signal does not affect the values of the Fourier series coefficients because these two graphs are exactly the same.

Fig. 7 shows the Fourier series coefficients of the $z_4(t)$ = Even{$x_3(t)$} signal.

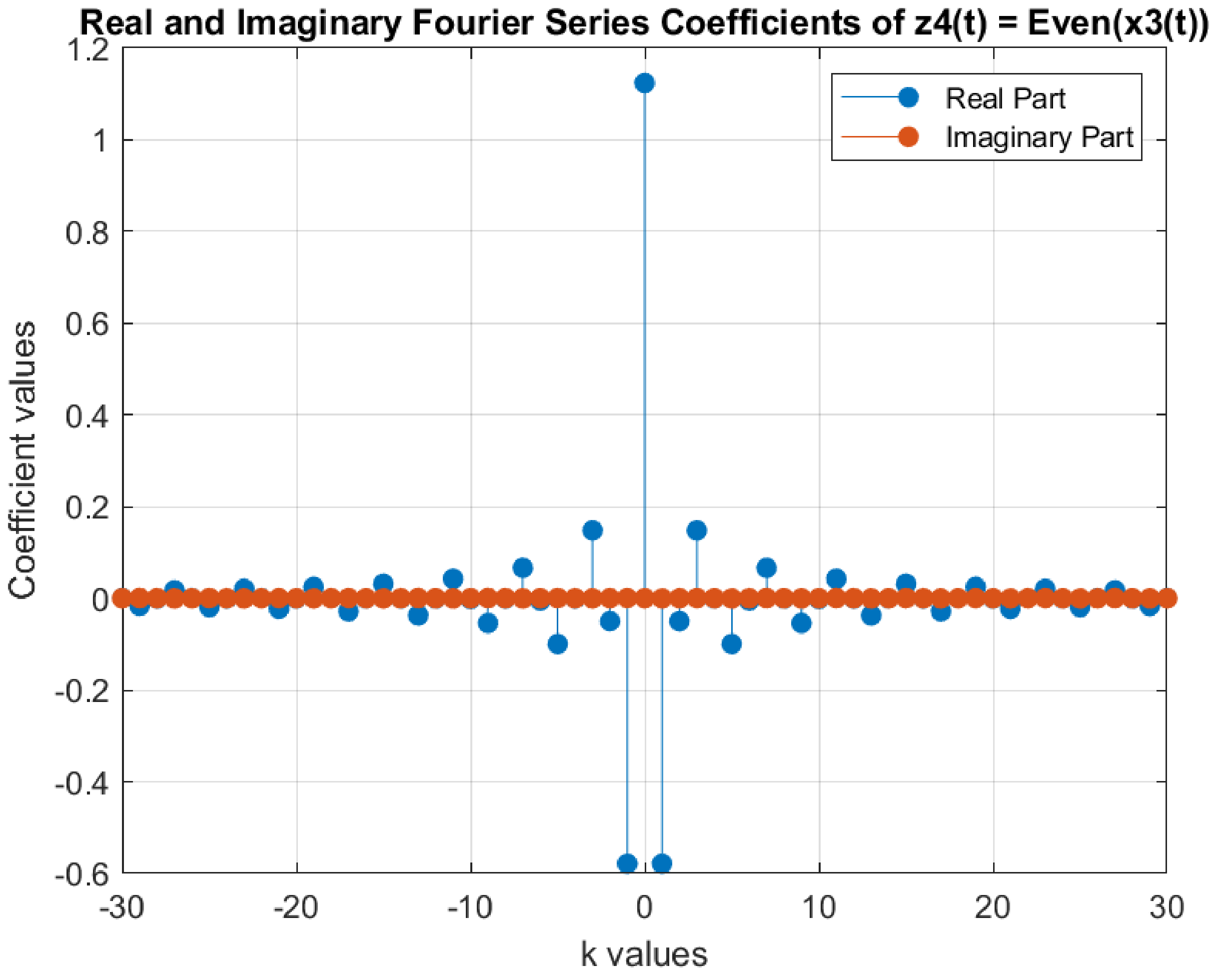

Fig. 7: Fourier series coefficients of the $z_4(t)$ signal

Even{$x_3(t)$} signal can be written as follows:

$$\frac{x_3(t) + x_3(-t)}{2} = \frac{x_3(t) + z_1(t)}{2}$$

As seen from the graph, even function of the $x_3(t)$ signal makes all imaginary valued Fourier series coefficients zero.

Finally, Fig. 8 shows the Fourier series coefficients of the $z_5(t) = x_3(t)^2$ signal.

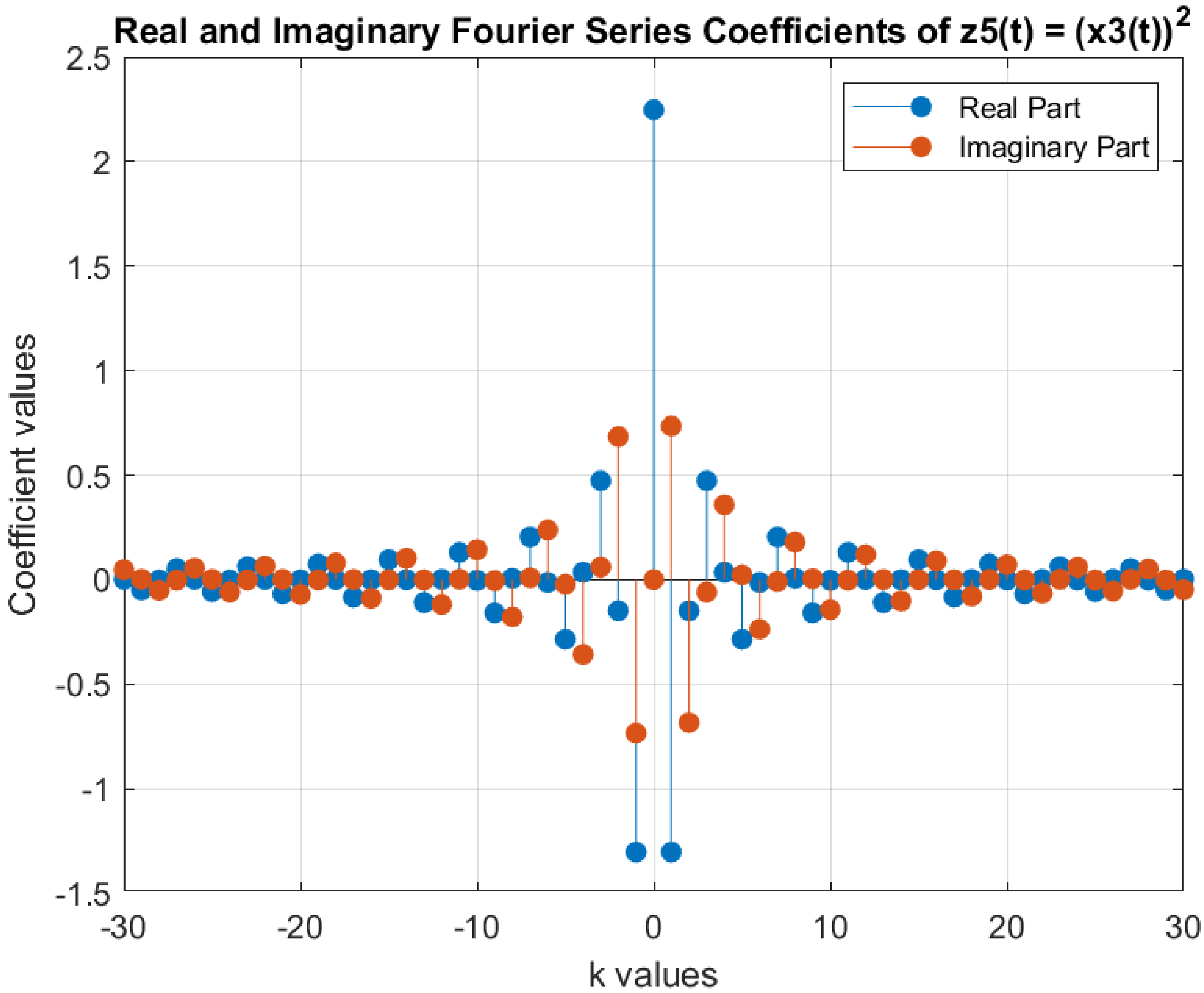

Fig. 8: Fourier series coefficients of the $z_5(t)$ signal

Again this graph also resembles to $x_3(t)$ graph, only near the origin point there are some differences in the values of real and imaginary parts of the Fourier series coefficients.

Part 3.1 A Second-Order System

The derivations for this part can be found at the end of this report.

Part 3.2 Implementation of the Second-Order System

In this part, calculations done in Part 3.1 are going to be implemented in MATLAB. Using backward approximation for first and second derivatives, system is rewritten in terms of y[n] and $x_3[n]$. Also system is assumed to be initially at rest therefore in the MATLAB code an array zeros is created. Below equation (Eqn. 1) shows the relation between y[n] and a combination of it's previous values and $x_3[n]$.

$$y[n] = \frac{x_3[n]+\left(\frac{2*M}{{T_S}^2}+\frac{c}{T_S}\right)*y[n-1]-\left(\frac{M}{{T_S}^2}\right)*y[n-2]}{\frac{M}{{T_S}^2}+\frac{c}{T_S}+\kappa} \qquad \text{(Eqn. 1)}$$

Fig. 9 is the graph of $x_3[n]$ sequence and following that Fig. 10 shows the y[n] sequence.

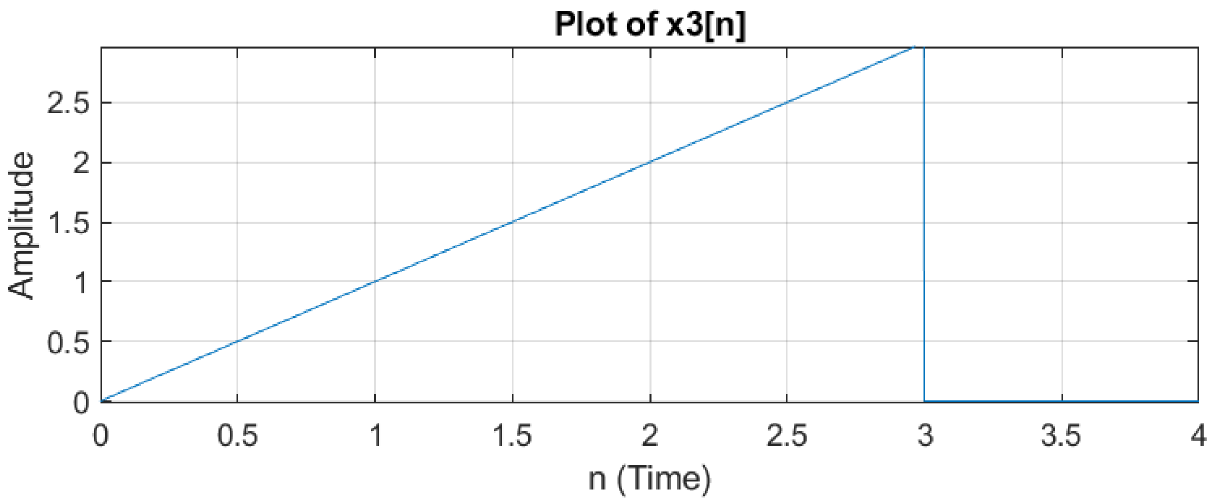

Fig. 9: Plot of the $x_3[n]$ sequence

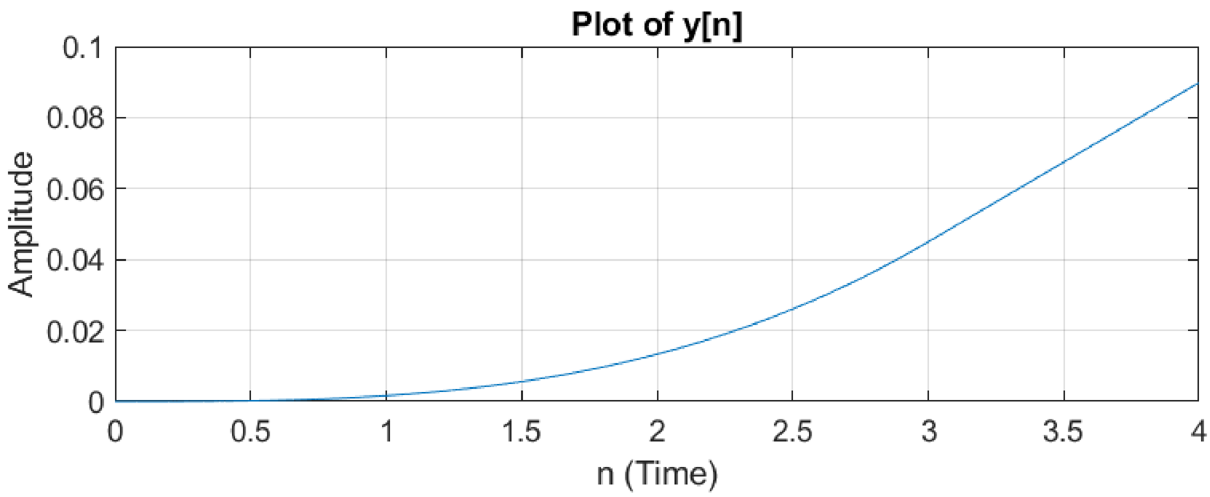

Fig. 10: Plot of the y[n] sequence

Using FSAnalysis function, Fourier series of both $x_3(t)$ and $y(t)$ signals are found. Fig.11 and Fig. 12 shows the real and imaginary Fourier series coefficients of the $x_3(t)$ respectively.

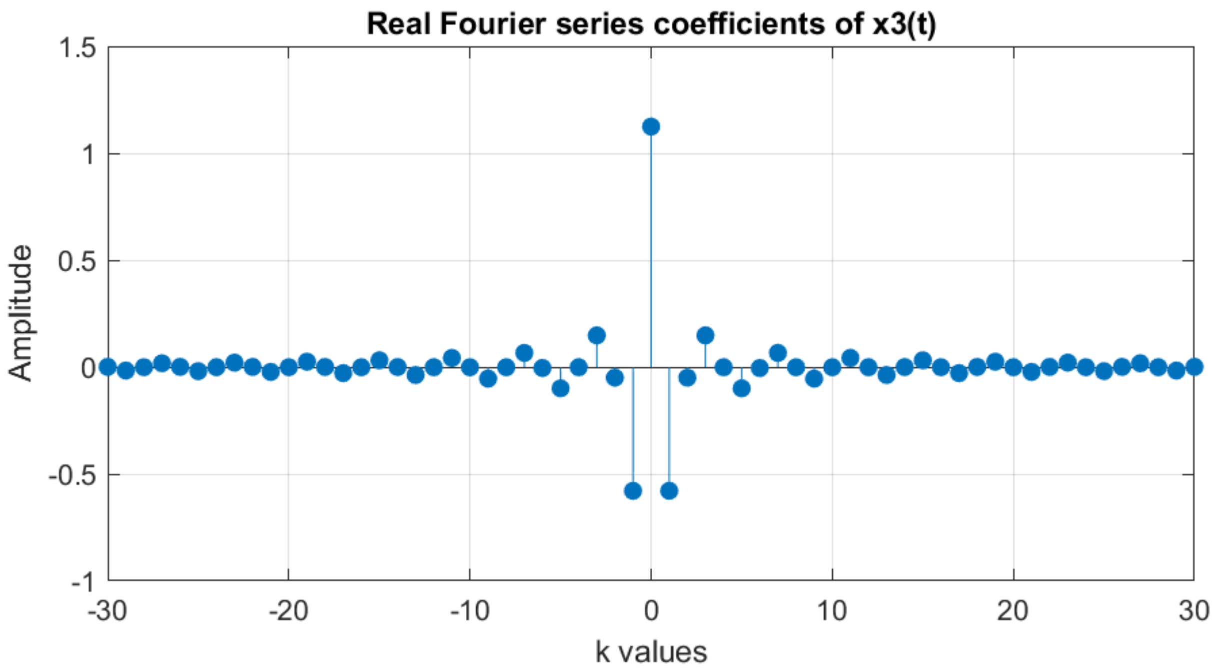

Fig. 11: Real Fourier series coefficients of $x_3(t)$ signal

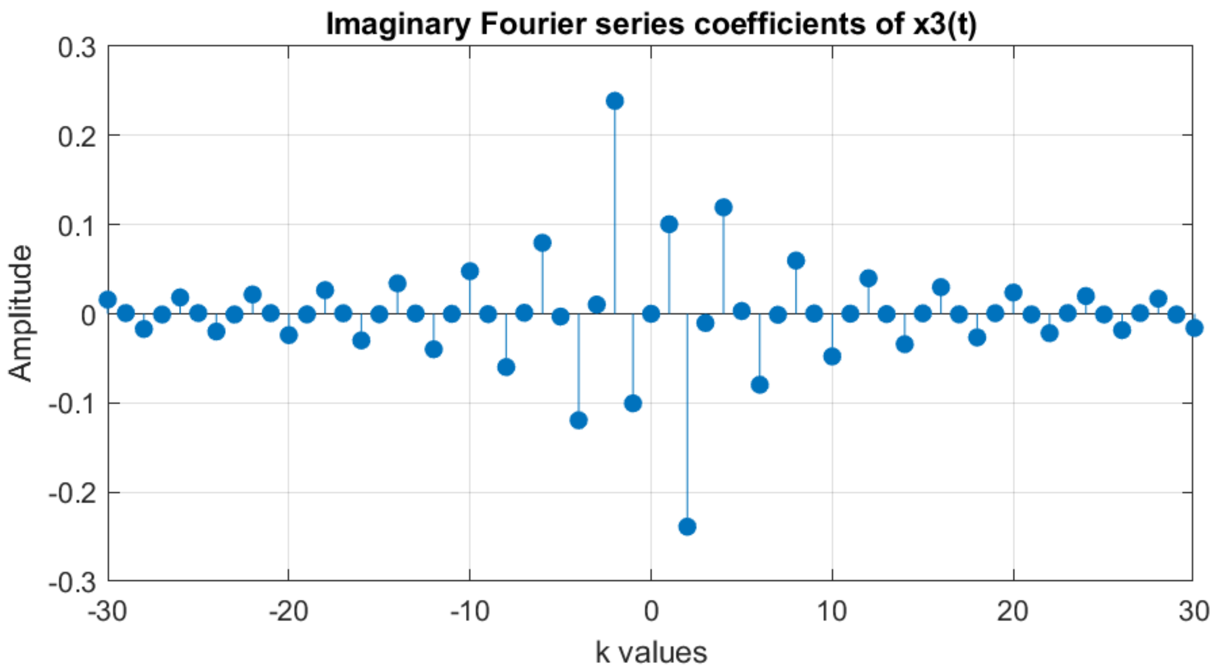

Fig. 12: Imaginary Fourier series coefficients of x3(t) signal

Fig. 13 and Fig. 14 shows the real and imaginary part of the Fourier series coefficients of y(t) respectively.

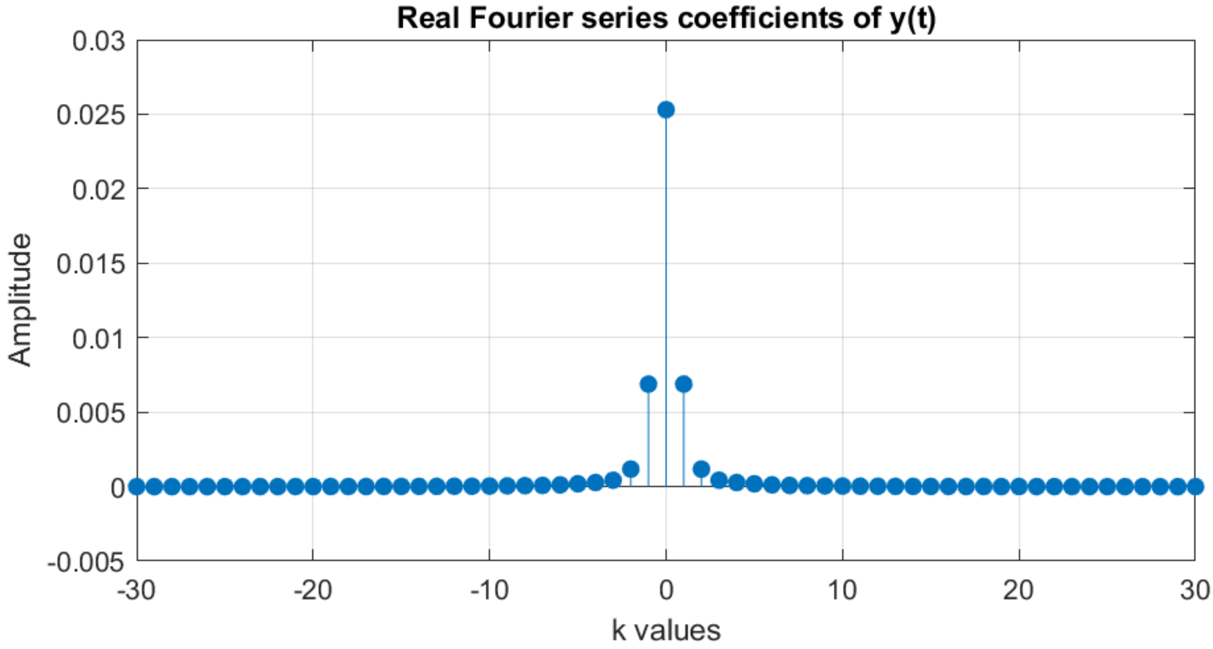

Fig. 13: Real Fourier series coefficients of y(t) signal

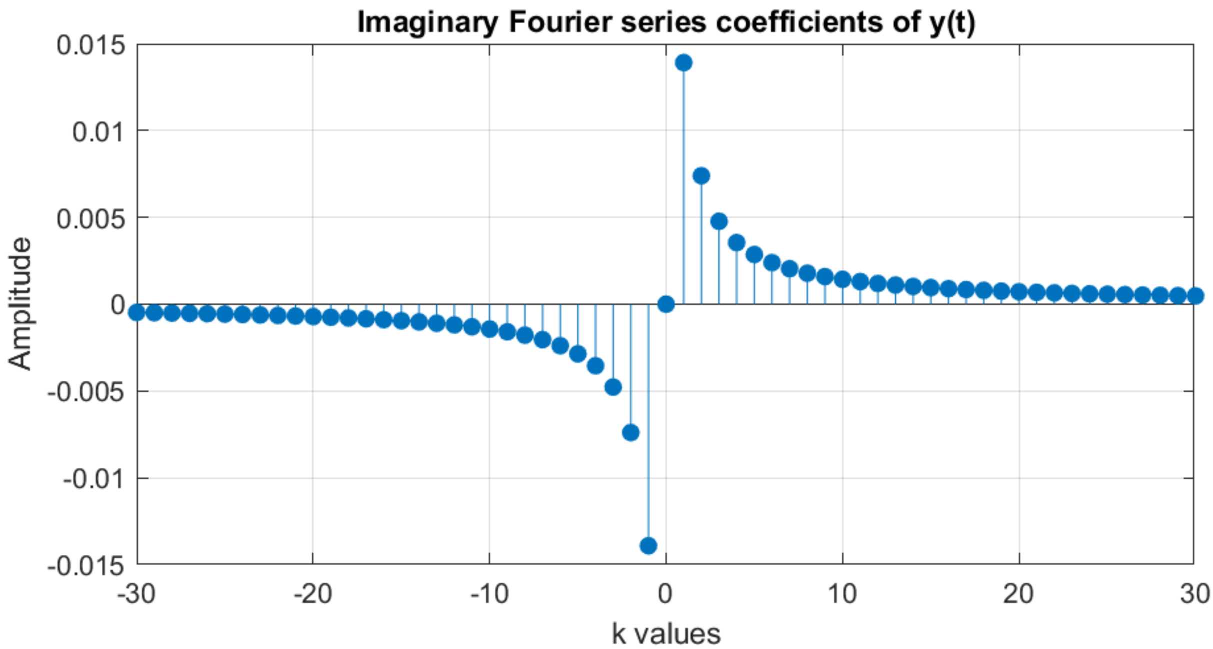

Fig. 14: Imaginary Fourier series coefficients of y(t) signal

Both real and imaginary Fourier series coefficients of y(t) signal are close to zero. Compared to x_3(t) Fourier series coefficient values, they are relatively lower can be acceptable as zero.

Appendix

FSAnalyisis function

```
function [fsCoeffs] = FSAnalysis(x, k)
N = length(x);
fsCoeffs = zeros(1, 2* k + 1);
for n = -k : k
    sum = 0;
    for m = 1:N
        sum = sum + x(m) * exp(1i * 2 * pi * n * (m-1) / N);
    end
    fsCoeffs(n + k + 1) = sum / N;
end
end
```

Part 1

```
Ts = 0.001;
k = 30;
t1 = 0:Ts:1;
int = -k:k;
%fundamental period of x1 is found as 2.
x1 = 8 * cos(10 * pi * t1) + 20 * sin(6 * pi * t1) - 11 * cos(30 * pi * t1);
fs_coefx1 = FSAnalysis(x1, k);

subplot(2,1,1);
stem(int, real(fs_coefx1), 'filled');
hold on;
stem(int, imag(fs_coefx1), 'filled');
title('Fourier Series Coefficients of x1(t)');
ylabel('Coefficient valeus');
xlabel('k values')
legend('Real Part', 'Imaginary Part');

t2 = -1:Ts:1;
x2 = exp((-t2));
fs_coefx2 = FSAnalysis(x2, k);

subplot(2,1,2);
stem(int, real(fs_coefx2), 'filled');
hold on;
stem(int, imag(fs_coefx2), 'filled');
title('Fourier Series Coefficients of x2(t)');
xlabel('k values')
ylabel('Coefficient values');
legend('Real Part', 'Imaginary Part');


sampled_x1 = sum(abs(x1).^2) * Ts;
%parseval's eqn.
par_x1 = sum(abs(fs_coefx1).^2);
disp(['Time-domain energy of x1: ', num2str(sampled_x1)]);
disp(['Frequency-domain energy of x1: ', num2str(par_x1)]);
```

Part 2

```
Ts = 0.001;
t = 0:Ts:4;
k = 30;
int = -k:k;

ramp = t;
u3 = double(t >= 3);
x3 = ramp - [zeros(1,3000), ramp(1:end-3000)] - 3*u3;

fs_coefx3 = FSAnalysis(x3, k);

figure;
stem(int, real(fs_coefx3), 'filled');
hold on;
stem(int, imag(fs_coefx3), 'filled');
title('Real and Imaginary Fourier Series Coefficients of x3(t)');
xlabel('k values');
ylabel('Coefficient values');
grid on;
legend('Real part', 'Imaginary part')

%--------------------------------------------------------------
z1 = fliplr(x3);
fs_coefz1 = FSAnalysis(z1, k);

figure;
stem(int, real(fs_coefz1), 'filled');
hold on;
stem(int, imag(fs_coefz1), 'filled');
title('Real and Imaginary Fourier Series Coefficients of z1(t) = x3(-t)');
xlabel('k values');
ylabel('Coefficient values');
grid on;
legend('Real Part', 'Imaginary Part')

%--------------------------------------------------------------
z2 = 1 * (t >= 0) - 1 * (t >= 3) - 3 * (t == 3);
fs_coefz2 = FSAnalysis(z2, k);

figure;
stem(int, real(fs_coefz2), 'filled');
hold on;
stem(int, imag(fs_coefz2), 'filled');
title('Real and Imaginary Fourier Series Coefficients of z2(t) = d(x2(t))/dt');
xlabel('k values');
ylabel('Coefficient values');
grid on;
legend('Real Part', 'Imaginary Part')

%--------------------------------------------------------------
t_shifted = t - 2;
z3 = (t).*(t_shifted >= -2) - (t-3).*(t_shifted >= 1) - 3 * (t_shifted >= 1);

fs_coefz3 = FSAnalysis(z3, k);

figure;
stem(int, real(fs_coefz3), 'filled');
```

```
hold on;
stem(int, imag(fs_coefz3), 'filled');
title('Real and Imaginary Fourier Series Coefficients of z3(t) = x3(t+2)');
xlabel('k values');
ylabel('Coefficient values');
grid on;
legend('Real Part', 'Imaginary Part')

%----------------------------------------------------------------
%Evenfunc = [x(t)+x(-t)]/2
z4 = (x3 + z1)/2;

fs_coefz4 = FSAnalysis(z4, k);

figure;
stem(int, real(fs_coefz4), 'filled');
hold on;
stem(int, imag(fs_coefz4), 'filled');
title('Real and Imaginary Fourier Series Coefficients of z4(t) = Even(x3(t))');
xlabel('k values');
ylabel('Coefficient values');
grid on;
legend('Real Part', 'Imaginary Part')

%----------------------------------------------------------------
z5 = x3.*x3;

fs_coefz5 = FSAnalysis(z5, k);

figure;
stem(int, real(fs_coefz5), 'filled');
hold on;
stem(int, imag(fs_coefz5), 'filled');
title('Real and Imaginary Fourier Series Coefficients of z5(t) = (x3(t))^2');
xlabel('k values');
ylabel('Coefficient values');
grid on;
legend('Real Part', 'Imaginary Part')
```

Part 3

```
M = 100;
c = 0.1;
Kappa = 0.1;
k = 30;
Ts = 0.001;
t = 0:Ts:4;
int = -k:k;

r_t = t;
u_t_3 = double(t >= 3);
x3 = r_t - [zeros(1,3000), r_t(1:end-3000)] - 3*u_t_3;

y = zeros(size(t));
denumerator = M/Ts^2 +c/Ts + Kappa;

for n = 3:length(x3)
    y(n) = (x3(n) +(2*M/Ts^2 +c/Ts) * y(n-1) - (M/Ts^2) * y(n-2)) / denumerator;
```

```
end

figure;
subplot(211)
plot(t, x3)
title('Plot of x3[n]')
ylabel('Amplitude')
xlabel('n (Time)')
grid on;

subplot(212)
plot(t, y)
title('Plot of y[n]')
ylabel('Amplitude')
xlabel('n (Time)')
grid on;


fs_coefx3 = FSAnalysis(x3, k);
fs_coefy = FSAnalysis(y, k);

figure;
subplot(221);
stem(int, real(fs_coefx3), 'filled');
title('Real Fourier series coefficients of x3(t)');
xlabel('k values');
ylabel('Amplitude');
grid on;

subplot(222);
stem(int, imag(fs_coefx3), 'filled');
title('Imaginary Fourier series coefficients of x3(t)');
xlabel('k values');
ylabel('Amplitude');
grid on;


subplot(223);
stem(int, real(fs_coefy), 'filled');
title('Real Fourier series coefficients of y(t)');
xlabel('k values');
ylabel('Amplitude');
grid on;

subplot(224);
stem(int, imag(fs_coefy), 'filled');
title('Imaginary Fourier series coefficients of y(t)');
xlabel('k values');
ylabel('Amplitude');
grid on;
```

Part 1.2)

$x_1(t) = 8\cos(10\pi t) + 20\sin(6\pi t) - 11\cos(30\pi t)$

$\quad w=10\pi \qquad w=6\pi \qquad w=30\pi$

$gcd(10\pi, 6\pi, 30\pi) = 2\pi = w_0 \Rightarrow T_0 = 1$

$$x_1(t) = 4(\underbrace{e^{j10\pi t}}_{a_5} + \underbrace{e^{-j10\pi t}}_{a_{-5}}) + \frac{10}{j}(\underbrace{e^{j6\pi t}}_{a_3} - \underbrace{e^{-j6\pi t}}_{a_{-3}}) - \frac{11}{2}(\underbrace{e^{j30\pi t}}_{a_{15}} + \underbrace{e^{-j30\pi t}}_{a_{-15}})$$

$a_3 = \frac{10}{j} \qquad a_{-3} = \frac{-10}{j}$

$a_5 = 4 \qquad a_{-5} = 4$

$a_{15} = \frac{-11}{2} \qquad a_{-15} = \frac{-11}{2}$

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Part 1.2)

$x_2(t) = e^{-t}$ for $-1 < t < 1$, $T = 2s$

$a_0 = \frac{1}{2}\int_{-1}^{1} e^{-t}\,dt = \frac{1}{2} -e^{-t}\Big|_{-1}^{1} = \frac{1}{2}\left(-e^{-1} + e^{1}\right) = 1.1752$

using symbolab

$a_1 = \frac{1}{2}\int_{-1}^{1} e^{-t}\, e^{-j\pi t}\,dt = \frac{1}{2}\cdot\left(\frac{e^{-(1+j\pi)t} - e^{(1+j\pi)t}}{1+j\pi}\right) \approx 0.1 - j0.32$

complex conjugate

$a_{-1} = a_1^{*} = 0.1 + j0.32$

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Part 3.1

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jkw_0t} \qquad \frac{dy(t)}{dt} = (jw_0)\,Y(jw_0) \qquad \frac{d^2y(t)}{dt^2} = (jw_0)^2\,Y(jw_0)$$

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0t}$$

$$b_k = \frac{1}{T_0}\int_0^{T_0} y(t)\, e^{-jkw_0t}\, dt$$

$$\Rightarrow M\sum_{k=-\infty}^{\infty} -w_0^2\, b_k e^{jkw_0t} + c\sum_{k=-\infty}^{\infty} (jw_0)\, b_k e^{jkw_0t} + K\sum_{k=-\infty}^{\infty} b_k e^{jkw_0t} = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0t}$$

$$\Rightarrow a_k = (-Mw_0^2 + c\cdot(jw_0) + K)\, b_k$$

$$H(jw_0) = \frac{F(jw_0)}{Y(jw_0)} = \frac{(-Mw_0^2 + c(jw_0) + K)\, b_k}{b_k} = -Mw_0^2 + c(jw_0) + K$$