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Department: EEE

Course Code: EEE321

Section: 02

Experiment Number: 05

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Lab 5 Report

Introduction:

This lab aims to finding equivalent impulse response of systems using Fourier transform. In the last part of the experiment, the location of an object will be estimated using a trasmitter-receiver (T/R), and use this information with Doppler effect to estimate the velocity of a moving object.

Analysis:

Part 1

Derivation for this part can be found at the end of this report.

Part 2.1 Implementing the Fourier Transform

A function calculating Fourier Transform of a given signal is written in MATLAB. The code can be seen at the end.

Part 2.2 Testing the Function

Derivation of this part can be found at the end. The function is tested with input signal x(t) = cos(2π30t).

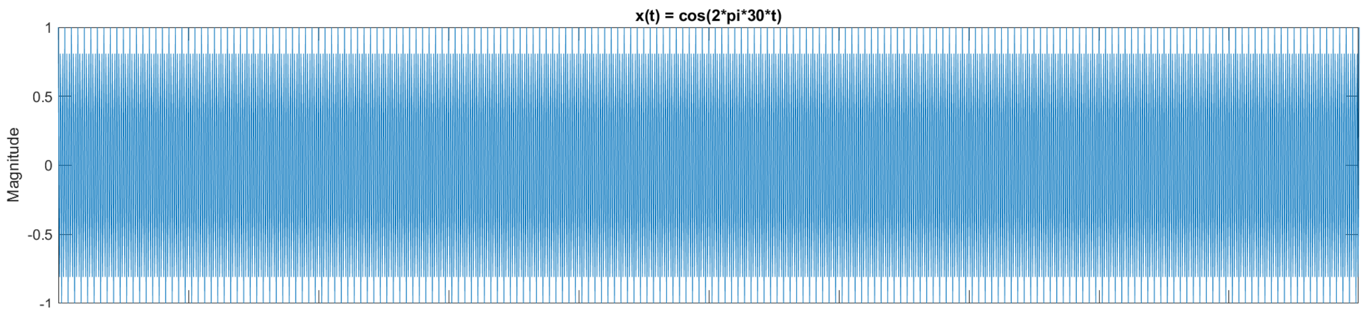


Fig. 1: Plot of x(t)

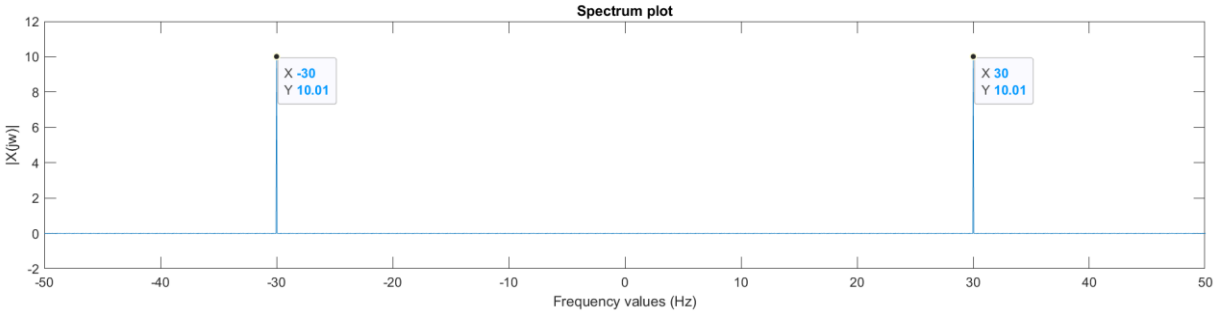


Fig. 2: Spectrum plot of x(t)

As it can be seen from Fig. 2, spectrum plot has two peak points, which are at -30Hz and 30Hz. Desired result is obtained.

Part 3.1 Derivation of Relations

Derivations for this part can be seen at the end of this report.

Part 3.2 Estimating Distances

From 3.1 derivations, $r_1(t)$, $r_2(t)$ equations are given below for this part.

$$r_1(t) = x_1\left(t - \frac{2 * d_1}{c}\right) + x_2\left(t - \frac{d_1 + d_2}{c}\right)$$

$$r_2(t) = x_1\left(t - \frac{d_1 + d_2}{c}\right) + x_2\left(t - \frac{2 * d_2}{c}\right)$$

Plots of $x_1(t)$, $x_2(t)$, $r_1(t)$, and $r_2(t)$ are given below respectively.

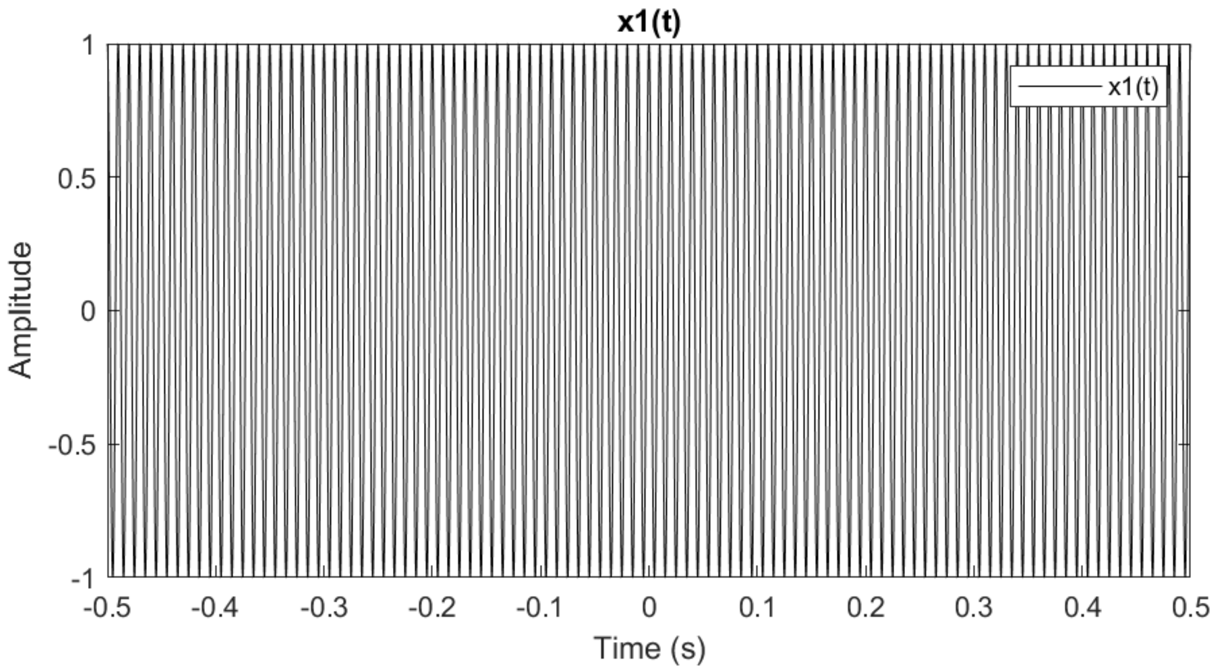


Fig. 3: Plot of x1(t)

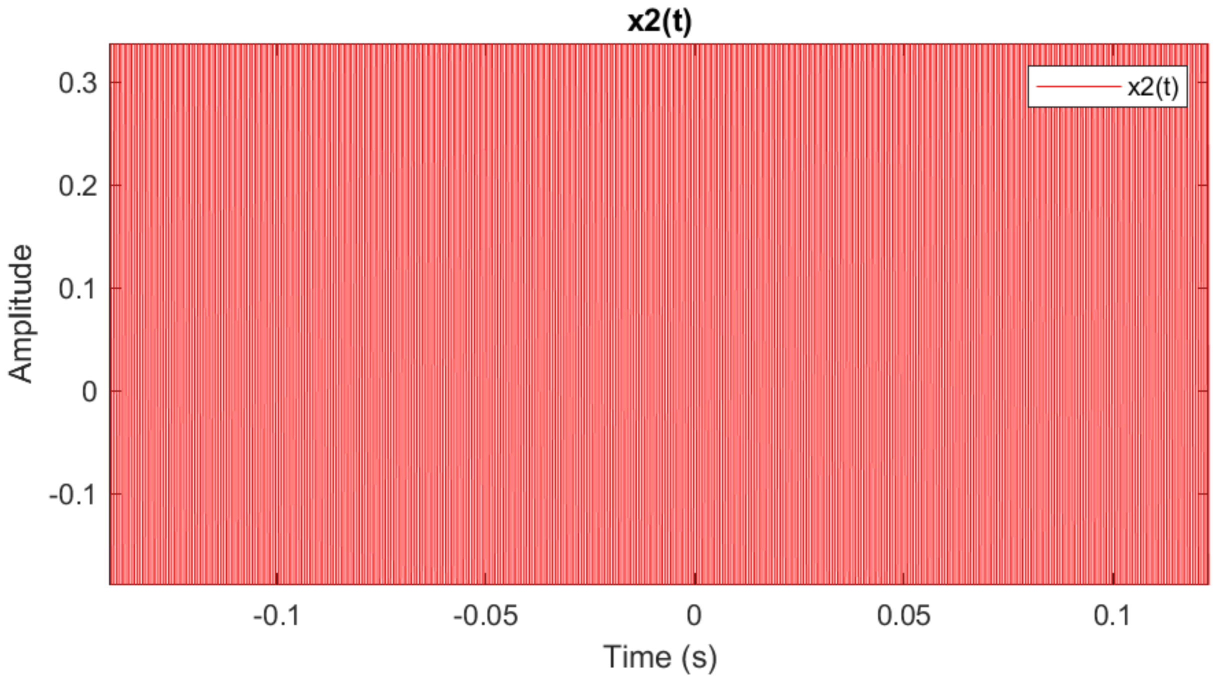


Fig. 4: Plot of x2(t)

Because f2 = 800Hz, plot of $x_2(t)$ is compressed.

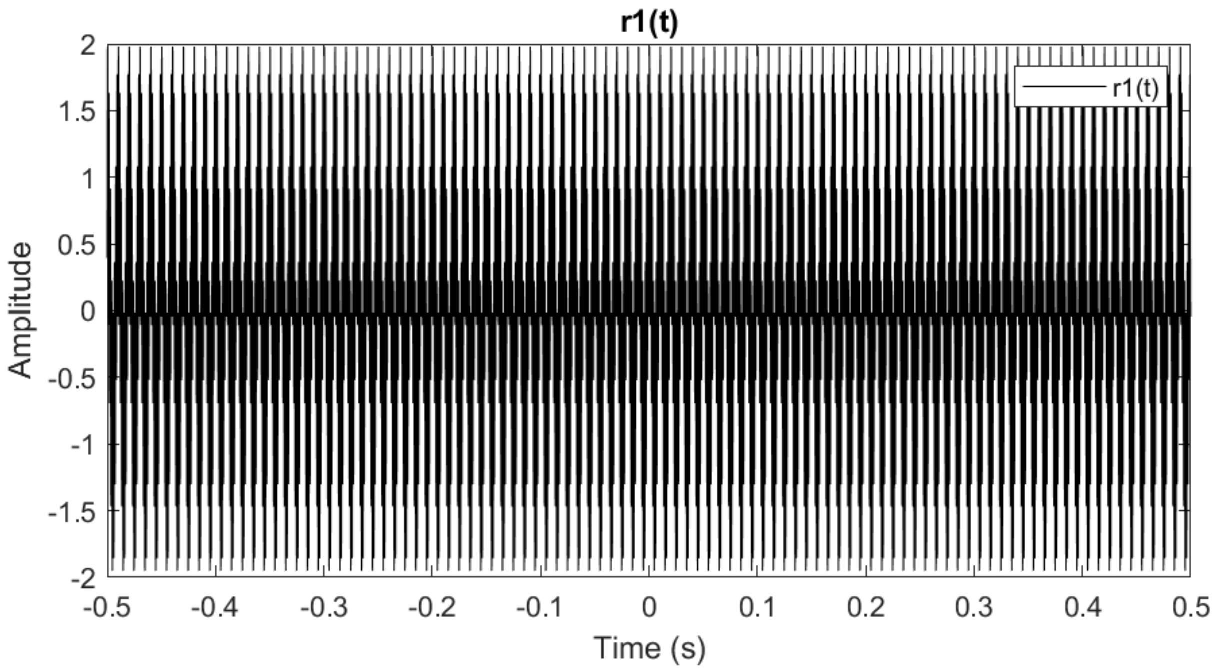


Fig. 5: Plot of r1(t)

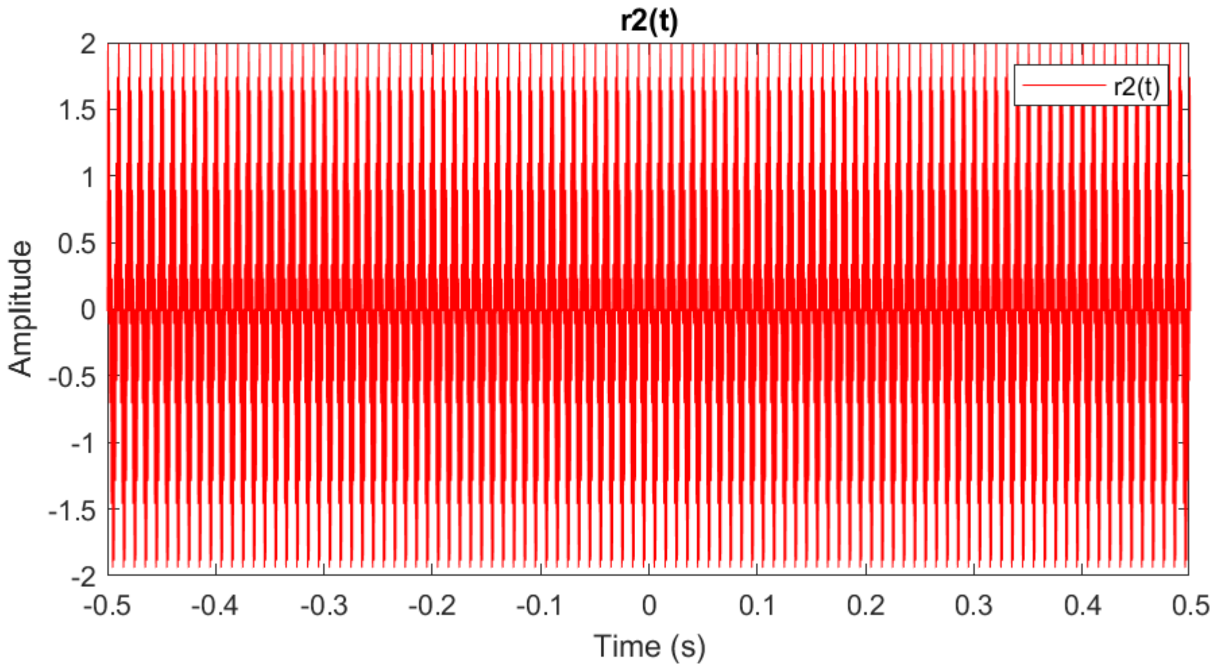


Fig. 6: Plot of r2(t)

Since both $r_1(t)$ and $r_2(t)$ equations are linear combination of $x_1(t)$ and $x_2(t)$ signals, $R_1(jw)$ and $R_2(jw)$ have peaks at 100Hz and 800Hz.

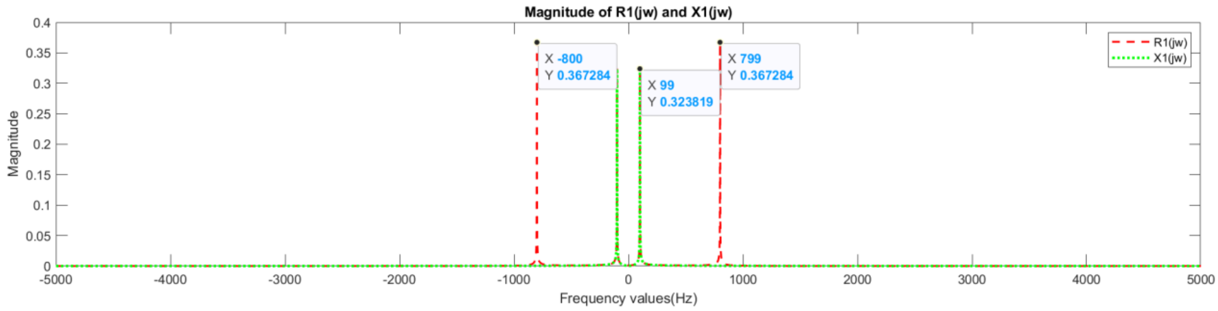


Fig. 7: Plot of R1(jw) and X1(jw)

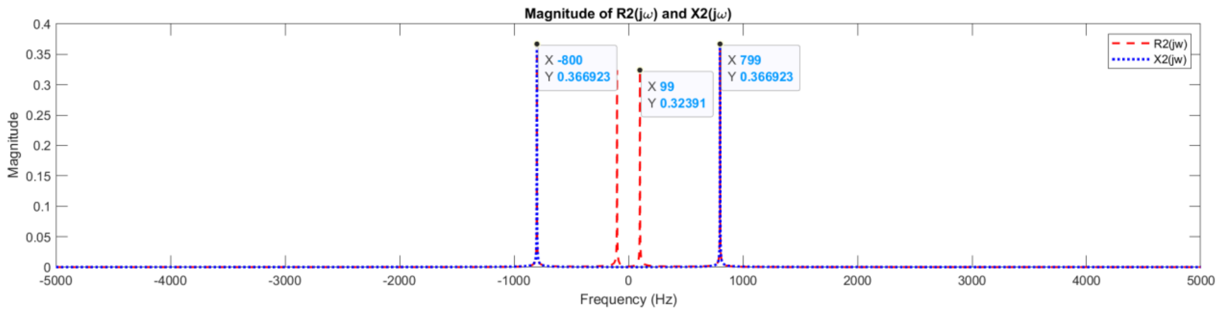


Fig. 8: Plot of R2(jw) and X2(jw)

From R1(jw) and R2(jw), applying ideal band-pass filter, Y1(jw) and Y2(jw) signals are obtained.

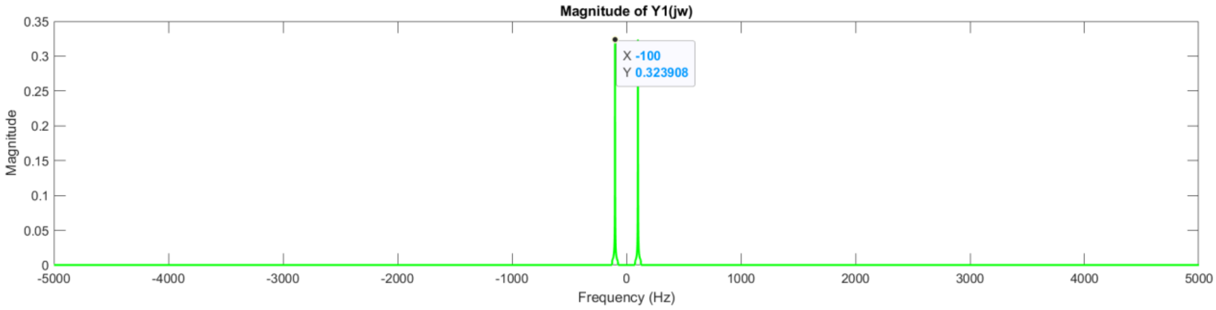


Fig. 9: Plot of Y1(jw)

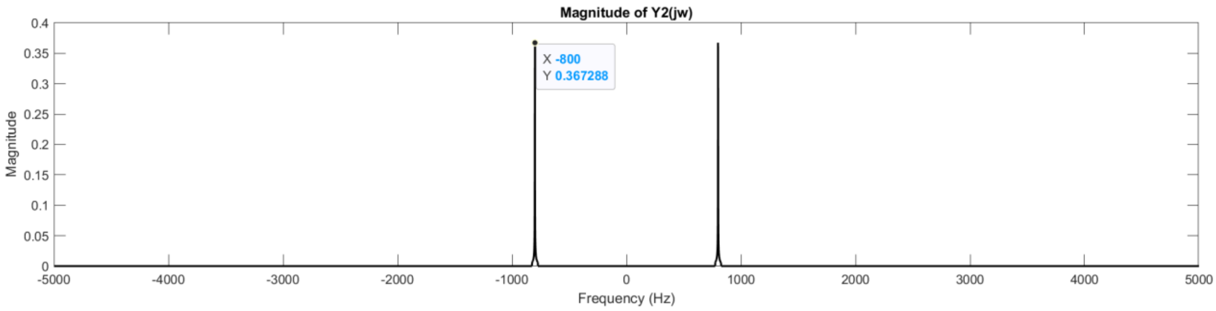


Fig. 10: Plot of Y2(jw)

Estimated distance d1: 0.049438 meters

Estimated distance d2: 0.099993 meters

True distance d1: 0.050000 meters

True distance d2: 0.100000 meters

Here, the estimated and true d1, d2 distance can be seen. Estimated values are nearly same with true distance values with a slight difference.

Part 4

Derivations for this part can be found at the end of the report.

Equations of r_1(t) and r_2(t) signals fort his part are given below.

$$r_1(t) = x_1\left(\left(\frac{c+v_1}{c-v_1}\right) * \left(t - \frac{2 * d_1}{c}\right)\right) + x_2\left(\left(\frac{c+v_2}{c-v_2}\right) * \left(t - \frac{d_1 + d_2}{c}\right)\right)$$

$$r_2(t) = x_1\left(\left(\frac{c+v_1}{c-v_1}\right) * \left(t - \frac{d_1 + d_2}{c}\right)\right) + x_2\left(\left(\frac{c+v_2}{c-v_2}\right) * \left(t - \frac{2 * d_2}{c}\right)\right)$$

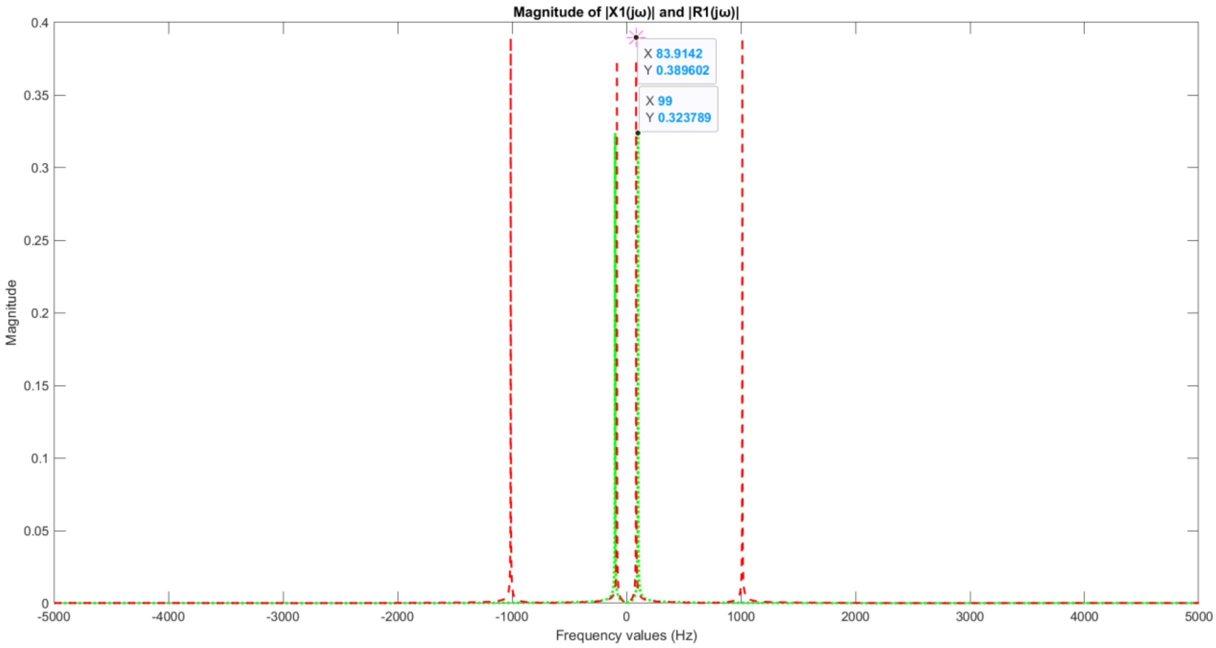


Fig. 11: Estimated frequency for X1(jw)

Estimated frequency is found as 83,914Hz.

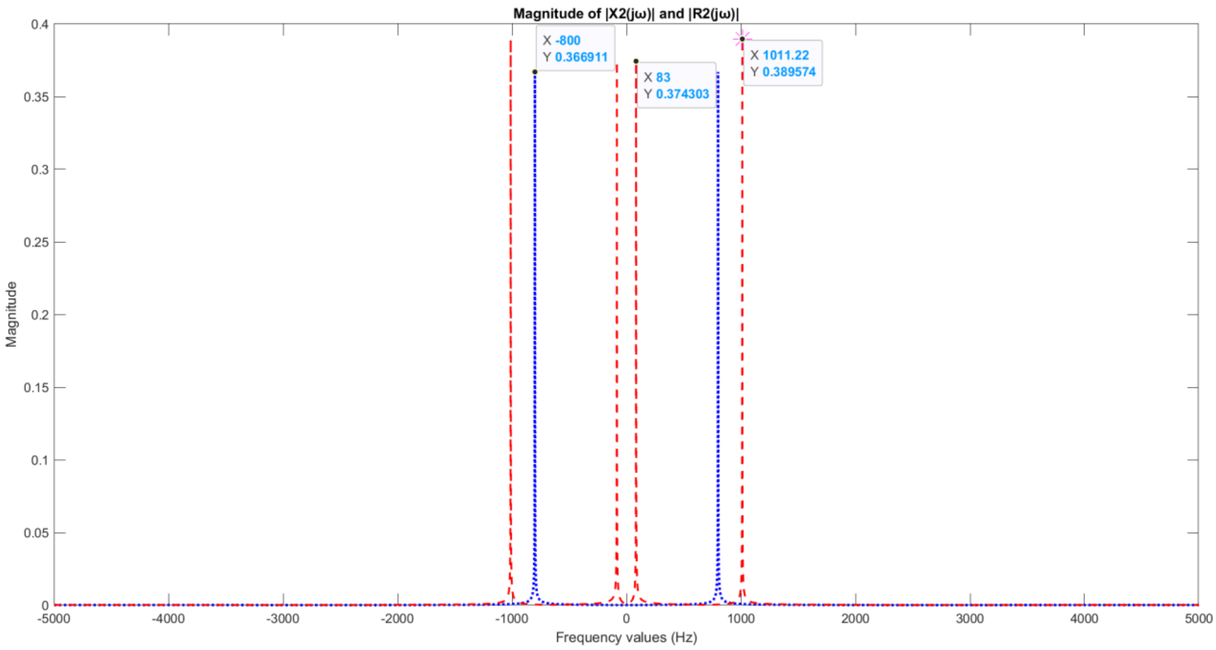


Fig. 12: Estimated frequency for X2(jw)

Estimated frequency is found as 1011,22Hz.

Here is th estimated velocity values:

Estimated v1: -30.0045m/s

Estimated v2: 39.9998m/s

True v1: -30m/s

True v2: 40m/s

Again, the estimated and true velocity values are nearly same with a slight difference.

Appendix

FoureirTransform

```
function [frequency_array] = FourierTransform(x, t, Ts)
    N = length(x);
    Fs = 1 / Ts;
    f = linspace(-Fs/2, Fs/2, N);
    X_omega = zeros(1, N);
    for k = 1:N
        omega = 2 * pi * f(k);
        X_omega(k) = sum(x .* exp(-1j * omega * t)) * Ts;
    end
    frequency_array = X_omega;
end
```

Part 2

```
Ts = 0.01;
t = -10:Ts:10;
x_t = cos(2 * pi * 30 * t);
X_omega = FourierTransform(x_t, t, Ts);
Fs = 1/Ts;
xomega_freq = linspace(-Fs/2, Fs/2, length(X_omega));

figure;
subplot(211)
plot(xomega_freq, X_omega)
title('Spectrum plot')
xlabel('Frequency values (Hz)')
ylabel('|X(jw)|')
subplot(212)
plot(t, x_t)
title('x(t) = cos(2*pi*30*t)')
xlabel('Time (s)')
ylabel('Magnitude')
xlim([-10 10])
```

Part 3

```
f1 = 100;
f2 = 800;
T = 1;
c = 343;
Ts = 0.0001;
Fs = 10000;
d1_true = 0.05;
d2_true = 0.1;

t = -T/2:1/Fs:T/2-Ts;
N = length(t);

pulse = zeros(size(t));
pulse(abs(t) <= T/2) = 1;

x1 = cos(2*pi*f1*t) .* pulse;
x2 = cos(2*pi*f2*t) .* pulse;
```

```
r1 = (cos(2*pi*f1*(t - 2*d1_true/c)) + cos(2*pi*f2*(t - (d1_true+d2_true)/c))) .*
pulse;
r2 = (cos(2*pi*f1*(t - (d1_true+d2_true)/c)) + cos(2*pi*f2*(t - 2*d2_true/c))) .*
pulse;

X1_omega = FourierTransform(x1, t, Ts);
X2_omega = FourierTransform(x2, t, Ts);
R1_omega = FourierTransform(r1, t, Ts);
R2_omega = FourierTransform(r2, t, Ts);

f = (-N/2:N/2-1)*(1/(Ts*N));

figure;
subplot(211);
plot(f, abs(R1_omega), 'r--', 'LineWidth', 1.5, 'DisplayName', 'R1(jω)');
hold on;
plot(f, abs(X1_omega), 'g:', 'LineWidth', 2, 'DisplayName', 'X1(jω)');
title('Magnitude of R1(jw) and X1(jw)');
xlabel('Frequency values(Hz)');
ylabel('Magnitude');
legend('R1(jw)', 'X1(jw)');
hold off;

subplot(2,1,2);
plot(f, abs(R2_omega), 'r--', 'LineWidth', 1.5, 'DisplayName', 'R2(jω)');
hold on;
plot(f, abs(X2_omega), 'b:', 'LineWidth', 2, 'DisplayName', 'X2(jω)');
title('Magnitude of R2(j\omega) and X2(j\omega)');
xlabel('Frequency (Hz)');
ylabel('Magnitude');
legend('R2(jw)', 'X2(jw)');
hold off;

figure;

subplot(221);
plot(t, x1, 'k');
title('x1(t)');
ylabel('Amplitude');
xlabel('Time (s)');
legend('x1(t)');

subplot(2, 2, 2);
plot(t, x2, 'r');
title('x2(t)');
ylabel('Amplitude');
xlabel('Time (s)');
legend('x2(t)');

subplot(223);
plot(t, r1, 'k');
title('r1(t)');
ylabel('Amplitude');
xlabel('Time (s)');
legend('r1(t)');

subplot(224);
plot(t, r2, 'r');
```

```
title('r2(t)');
ylabel('Amplitude');
xlabel('Time (s)');
legend('r2(t)');

omega_pass = 50;
BPF1 = double(abs(f) >= (f1 - omega_pass/2) & abs(f) <= (f1 + omega_pass/2));
BPF2 = double(abs(f) >= (f2 - omega_pass/2) & abs(f) <= (f2 + omega_pass/2));

% Filtered signals in the frequency domain
Y1_omega = R1_omega .* BPF1;
Y2_omega = R2_omega .* BPF2;
figure;

subplot(211);
plot(f, abs(Y1_omega), 'g', 'LineWidth', 1.5);
title('Magnitude of Y1(jw)');
xlabel('Frequency (Hz)');
ylabel('Magnitude');

subplot(212);
plot(f, abs(Y2_omega), 'k', 'LineWidth', 1.5);
title('Magnitude of Y2(jw)');
xlabel('Frequency (Hz)');
ylabel('Magnitude');


    % Find indices for f1 and f2
    [~, f1_idx] = min(abs(f - f1));
    [~, f2_idx] = min(abs(f - f2));

    % Phase calculation at the frequencies f1 and f2
    phase_d1 = angle(Y1_omega(f1_idx));
    phase_d2 = angle(Y2_omega(f2_idx));

    % Unwrap the phase and ensure it's within the range of 0 to 2*pi
    phase_d1_unwrapped = unwrap([0, phase_d1]);
    phase_d2_unwrapped = unwrap([0, phase_d2]);

    % Use the second element of the unwrapped phase
    phase_d1 = phase_d1_unwrapped(2);
    phase_d2 = phase_d2_unwrapped(2);

    lambda1 = c / f1;
    lambda2 = c / f2;
    estimated_d1 = (2*pi - mod(phase_d1, 2*pi)) * lambda1 / (4 * pi);
    estimated_d2 = (2*pi - mod(phase_d2, 2*pi)) * lambda2 / (4 * pi);

    % Output results and comparison with true values
    fprintf('Estimated distance d1: %f meters\n', estimated_d1);
    fprintf('Estimated distance d2: %f meters\n', estimated_d2);
    fprintf('True distance d1: %f meters\n', d1_true);
    fprintf('True distance d2: %f meters\n', d2_true);
```

Part 4

```
d1_true = 0.05;
d2_true = 0.1;
c = 343;
v1 = -30;
v2 = 40;
f1 = 100;
f2 = 800;
T = 1;
Ts = 0.0001;
samp_period = 10000;
t = linspace(-T/2, T/2, samp_period);

x1_t = cos(2 * pi * f1 * t);
x2_t = cos(2 * pi * f2 * t);

f_r1 = f1 * (c + v1) / (c - v1);
f_r2 = f2 * (c + v2) / (c - v2);

r1_t = (cos(2 * pi * f_r1 * (t - 2*d1_true/c)) + cos(2 * pi * f_r2 * (t -
(d1_true+d2_true)/c)));
r2_t = (cos(2 * pi * f_r1 * (t - (d1_true+d2_true)/c)) + cos(2 * pi * f_r2 * (t -
2*d2_true/c)));

X1_omega = FourierTransform(x1_t, t, Ts);
X2_omega = FourierTransform(x2_t, t, Ts);
R1_omega = FourierTransform(r1_t, t, Ts);
R2_omega = FourierTransform(r2_t, t, Ts);

f = (-samp_period/2:samp_period/2-1)*(1/(Ts*samp_period));

figure;
plot(f, abs(X1_omega), 'g:', 'LineWidth', 2, 'DisplayName', 'X1(jω)');
hold on;
plot(f, abs(R1_omega), 'r--', 'LineWidth', 1.5, 'DisplayName', 'R1(jω)');
plot(f_r1, max(abs(R1_omega)), 'm*', 'MarkerSize', 15, 'DisplayName', 'Expected
fr1');
title('Magnitude of |X1(jω)| and |R1(jω)|');
xlabel('Frequency values (Hz)');
ylabel('Magnitude');

hold off;

figure;
plot(f, abs(X2_omega), 'b:', 'LineWidth', 2, 'DisplayName', 'X2(jω)');
hold on;
plot(f, abs(R2_omega), 'r--', 'LineWidth', 1.5, 'DisplayName', 'R2(jω)');
plot(f_r2, max(abs(R2_omega)), 'm*', 'MarkerSize', 15, 'DisplayName', 'Expected
fr2');
title('Magnitude of |X2(jω)| and |R2(jω)|');
xlabel('Frequency values (Hz)');
ylabel('Magnitude');

hold off;

fr1 = 100;
fp1 = 83.912;
v1_estimate = (c * (fp1 - fr1)) / (f1 + fp1);
```

```
fr2 = 800;
fp2 = 1011.22;
v2_estimate = (c * (fp2 - fr2)) / (fr2 + fp2);
disp(['Estimated v1: ', num2str(v1_estimate), 'm/s']);
disp(['Estimated v2: ', num2str(v2_estimate), 'm/s']);
```

from Table 4.2

Part 1

a) $h_1(t) = \frac{d}{dt}\left(\frac{\sin w_c t}{2\pi t}\right)$ $\quad y_1(t) = \frac{\sin w_c t}{\pi t} \Rightarrow Y_1(jw) = \begin{cases} 1, & |w| < w_c \\ 0, & \text{otherwise} \end{cases}$

differentiation

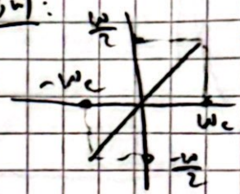
$\frac{1}{2}\frac{d}{dt}\left[\frac{\sin w_c t}{\pi t}\right] \Rightarrow h_1(t) = \frac{1}{2}\frac{d}{dt} y_1(t) \Rightarrow H_1(jw) = \frac{1}{2} jw\, Y_1(jw)$

$\Rightarrow H_1(jw) = \begin{cases} \frac{jw}{2}, & |w| < w_c \\ 0, & \text{otherwise} \end{cases}$

$[X(jw)\cdot H_1(jw) - X(jw)\cdot H_1(jw)\cdot H_2(jw)]\cdot H_3(jw)\cdot H_4(jw) = Y(jw)$

$H_{eq}(jw) = \frac{Y(jw)}{X(jw)} = H_3(jw)\cdot H_4(jw)[H_1(jw) - H_1(jw)\cdot H_2(jw)]$

$H_1(jw)$:



$|H_1(jw)| = \begin{cases} \frac{w}{2}, & |w| < w_c \\ 0 & \text{otherwise} \end{cases}$

$H_{eq}(jw) = H_1(jw)\cdot H_3(jw)\cdot H_4(jw) - H_1(jw)\cdot H_2(jw)\cdot H_3(jw)\cdot H_4(jw)$

Linearity

$h_2(t) = -\frac{1}{2} e^{jw_1 t} + \cos(w_1 t) \Rightarrow h_2(t) = -\frac{1}{2} y_2(t) + y_3(t) \Rightarrow H_2(jw) = -\frac{1}{2} Y_2(jw) + Y_3(jw)$

Table 4.2

$y_2(t) = e^{jw_1 t} \Rightarrow Y_2(jw) = 2\pi\,\delta(w - w_1)$

$H_2(jw) = \pi\,\delta(w + w_1)$

$y_3(t) = \cos(w_1 t) \Rightarrow Y_3(jw) = \pi[\delta(w - w_1) + \delta(w + w_1)]$

Table 4.2

$h_4(t) = u(t) \Rightarrow H_4(jw) = \frac{1}{jw} + \pi\,\delta(w)$

$H_{eq}(jw) = \frac{Y(jw)}{X(jw)} = H_1(jw)\cdot H_3(jw)\cdot H_4(jw) - H_1(jw)\cdot H_2(jw)\cdot H_3(jw)\cdot H_4(jw)$

$= \frac{jw}{2}\cdot e^{-j\frac{2\pi w}{w_2}}\cdot\frac{1}{jw} - \frac{jw}{2}\pi\,\delta(w + w_1)\cdot e^{-j\frac{2\pi w}{w_2}}\cdot\frac{1}{jw}$ $\quad$ if $|w| < w_c$

$= \frac{e^{-j\frac{2\pi w}{w_2}}}{2} - \frac{e^{j\frac{2\pi w_1}{w_2}}}{2}\pi$ $\quad$ if $|w| < w_c$ and $|w_1| < w_c$

$= \frac{e^{-j\frac{2\pi}{w_2}w}}{2}$ $\quad$ if $|w| < w_c$ and $|w_1| > w_c$

$= 0$ $\quad$ if $|w| > w_c$ and $|w_1| > w_c$

$$h_{eq}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_{eq}(j\omega)\, e^{j\omega t}\, dt$$

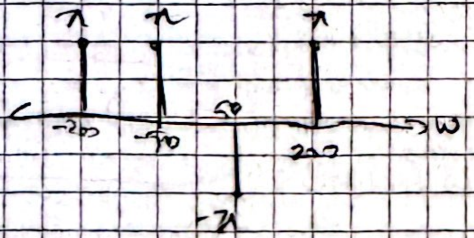
$$= \frac{1}{4\pi}\int_{-\omega_c}^{\omega}\left(e^{-j\frac{2\pi\omega}{\omega_2}} - e^{j\frac{2\pi\omega_1}{\omega_2}}\cdot\pi\right)e^{j\omega t}\,d\omega \quad \text{if } |\omega| < \omega_c \quad |\omega_1| < \omega_c$$

$$= \frac{1}{4\pi}\left[\int_{-\omega_c}^{\omega} e^{\left(jt - \frac{2\pi}{\omega_2}\right)\omega} - e^{j\frac{2\pi\omega_1}{\omega_2}}\cdot\pi\cdot\int_{-\omega_c}^{\omega_c} e^{j\omega t}\,d\omega\right]$$

$$= \frac{1}{4\pi}\left[\left(\frac{e^{\left(jt-\frac{2\pi}{\omega_2}\right)\omega}}{jt-\frac{2\pi}{\omega_2}}\right)\Bigg|_{-\omega_c}^{\omega_c} - e^{j\frac{2\pi\omega_1}{\omega_2}}\cdot\pi\cdot\frac{e^{j\omega t}}{jt}\Bigg|_{-\omega_c}^{\omega_c}\right]$$

$$= \frac{1}{4\pi}\left[\frac{e^{\left(jt-\frac{2\pi}{\omega_2}\right)\omega_c}}{jt-\frac{2\pi}{\omega_2}} - \frac{e^{\left(\frac{2\pi}{\omega_2}-jt\right)\omega_c}}{jt-\frac{2\pi}{\omega_2}} - e^{j\frac{2\pi\omega_1}{\omega_2}}\cdot\pi\left(\frac{e^{j\omega_c t} - e^{-j\omega_c t}}{jt}\right)\right]$$

$$x(t) = \sin(50t) + \cos(200t) \Rightarrow X(j\omega) = \frac{\pi}{j}\left[\delta(\omega-50) - \delta(\omega+50)\right] + \pi\left[\delta(\omega-200) + \delta(\omega+200)\right]$$



$$\boxed{Y(j\omega) = \frac{\pi}{2j}\left(e^{-j50} - e^{j50}\right) = -\sin(50)\,\pi}$$

$$H(j\omega) = \frac{e^{\frac{-j}{10}\omega}}{2} - \frac{e^{j\,7.5}}{2}\cdot\pi$$

$$\boxed{y(t) = -\sin(50)\,\pi\,\delta(t)}$$

b) $x(t)$ real, ...

$X(j\omega) = c$ for $\omega \in [0, 2\pi]$

$X(j\omega) = c\, e^{2\omega}$ for $\omega < -2\pi$

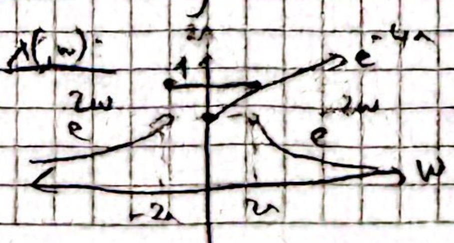
$$\int_{-\infty}^{\infty} x(t)\,dt = 1$$

$$X(0) = \int_{-\infty}^{\infty} x(t)\,dt = 1 \Rightarrow \underline{c = 1}$$

$$e^{-4\pi}\left(\frac{2e^{-2\pi j t} + 2e^{2\pi j t} - jt\,e^{-2\pi j t} + jt\,e^{2\pi j t}}{4+t^2}\right)$$

$$= e^{-4\pi}\left(\frac{e^{-2\pi j t}}{2+jt} + \frac{e^{2\pi j t}}{2-jt}\right)$$

$$\int_{-\infty}^{-2\pi} e^{2\omega}\, e^{j\omega t}\,d\omega = \frac{e^{\omega(2+jt)}}{(2+jt)}\Bigg|_{-\infty}^{-2\pi}$$



$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw)e^{jwt}\,dw = \frac{1}{2\pi}\left[\int_{-2\pi}^{2\pi} e^{jwt}\,dw + \int_{-\infty}^{-2\pi} e^{2w}\cdot e^{jwt}\,dw + \int_{2\pi}^{\infty} e^{-2w}e^{jwt}\,dw\right]$$

$$= \frac{1}{2\pi}\frac{e^{jwt}}{jt}\Big|_{-2\pi}^{2\pi} + \frac{1}{2\pi}\frac{e^{w(2+jt)}}{2+jt}\Big|_{-\infty}^{-2\pi} - \frac{1}{2\pi}\frac{e^{-w(2-jt)}}{2-jt}\Big|_{2\pi}^{\infty}$$

$$= \frac{1}{jt2\pi}\left[e^{j2\pi t} - e^{-j2\pi t}\right] + \frac{1}{(2+jt)2\pi}\cdot e^{-2\pi(2+jt)} + \frac{1}{(2-jt)2\pi}e^{-2\pi(2-j}$$

$$\Rightarrow x(t) = \frac{\sin 2\pi t}{\pi t} + \frac{e^{-4\pi}}{4+t^2}\left[4\cos(2\pi t) - 2\sin(2\pi t)\right]$$

Part 3:

$$r_1(t) = x_1\left(t - \frac{2d_1}{c}\right) + x_2\left(t - \frac{(d_1+d_2)}{c}\right), \quad r_2(t) = x_1\left(t - \frac{(d_1+d_2)}{c}\right) + x_2\left(t - \frac{2d_2}{c}\right)$$

$$R_1(jw) = X_1(jw)e^{-jw}\left(\frac{2d_1}{c}\right) + X_2(jw)e^{-jw}\left(\frac{d_1+d_2}{c}\right)$$

$$R_2(jw) = X(jw)e^{-jw}\left(\frac{d_1+d_2}{c}\right) + X_2(jw)e^{-jw}\left(\frac{2d_2}{c}\right)$$

$$y_1(t) = x_1\left(t - \frac{2d_1}{c}\right) \longrightarrow Y_1(jw) = X_1(jw)e^{-jw}\left(\frac{2d_1}{c}\right)$$

$$y_2(t) = x_2\left(t - \frac{2d_2}{c}\right) \longrightarrow Y_2(jw) = X_2(jw)e^{-jw}\left(\frac{2d_2}{c}\right)$$

$$-jw\frac{2d_1}{c} = \ln\left(\frac{Y_1(jw)}{X_1(jw)}\right) \qquad d_1 = \frac{jc}{2w}\ln\frac{Y_1(jw)}{X_1(jw)}$$

$$d_2 = \frac{jc}{2w}\ln\frac{Y_2(jw)}{X(jw)}$$

Part 4:

$$r_1(t) = x_1\left(\underbrace{\frac{c+v_1}{c-v_1}}_{\text{say } a}\left[t - \frac{2d_1}{c}\right]\right) + x_2\left(\underbrace{\frac{c+v_2}{c-v_2}}_{\text{say } b}\left[t - \frac{(d_1+d_2)}{c}\right]\right)$$

$$r_2(t) = x_1\left(a\left[t - \frac{(d_1+d_2)}{c}\right]\right) + x_2\left(b\left[t - \frac{2d_2}{c}\right]\right)$$

$$R_1(jw) = \frac{1}{|a|}X_1\left(\frac{jw}{a}\right)e^{-jw}\left(\frac{2d_1}{c}\right) + \frac{1}{|a|}X_2\left(\frac{jw}{a}\right)e^{-jw}\left(\frac{d_1+d_2}{c}\right)$$

$$R_2(jw) = \frac{1}{|b|}X_2\left(\frac{jw}{b}\right)e^{-jw}\left(\frac{d_1+d_2}{c}\right) + \frac{1}{|b|}X_2\left(\frac{jw}{b}\right)e^{-jw}\left(\frac{2d_2}{c}\right)$$