EEE342 Lab #2 Report

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1. Introduction

The purpose of this laboratory experiment is to perform system identification studies on a physical DC motor in frequency domain. In the first part, the bode plot of the estimated transfer function which is found in first experiment is used. Then, a sinusoidal input will be given to the system and corresponding bode plot will be analyzed. There will be in total 7 bode plot results, for each different angular frequency and simulation duration data. Finally, Pade' approximation is used to generate a new transfer function. In conclusion, the bode plots obtained in each part will be compared.

2. Laboratory Content

2.1 Q1: Bode plot generation

In the first part, it is wanted to use the estimated transfer function derived in the first lab. The derived transfer function is given below:

$$G(s) = \frac{13.42}{0.22s + 1}$$

So, using this estimated transfer function, the bode plot of the DC motor will be plotted. In Fig. 1, both magnitude and phase bode plots are depicted.

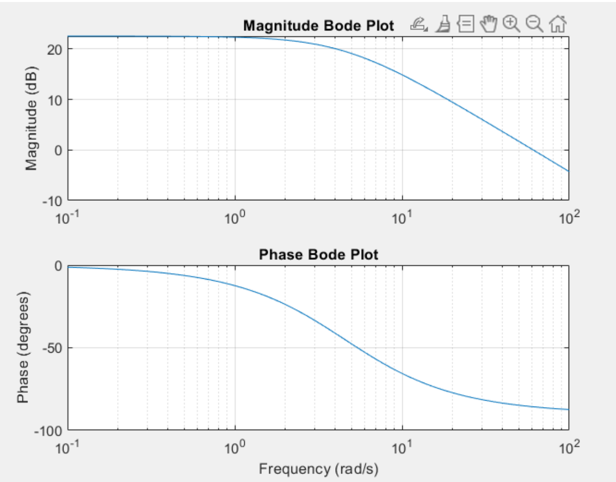

Fig. 1: Bode plot of the estimated transfer function of DC motor derived in Lab #1.

2.2 Q2: Applying sinusoidal inputs

In this step, a sinusoidal input is applied with 7 different angular frequency values following different time durations. The purpose is to compare the output and input signals on the same plot. The input signal is given below:

$$x(t) = 10\sin(\omega t)$$

The list of asked angular frequency and time durations are given in table below:

Table 1: Table of asked angular frequency and time duration

Angular Frequency (rad/s)	Time Duration (s)
0.1	70
0.3	70
1	25
3	25
10	10
30	10
100	10

Staring from Fig. 2, the angular frequency values listed above is inputted to the input signal, and output signal is observed on the same plot with the input signal, until Fig. 8 (included).

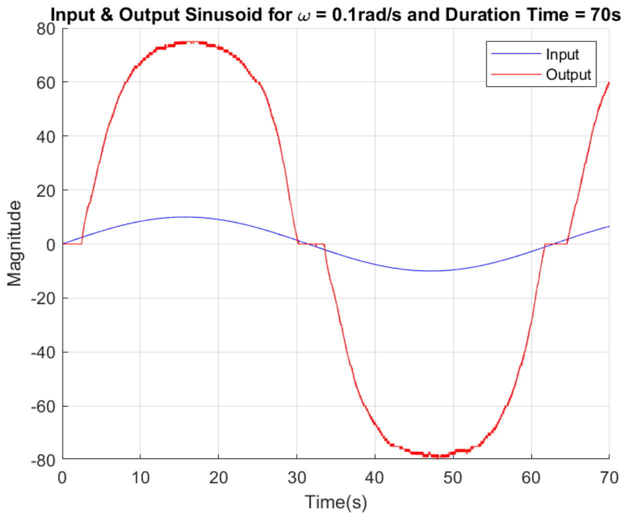

Fig. 2: Input and output signals (ω = 0.1 rad/s and t = 70s)

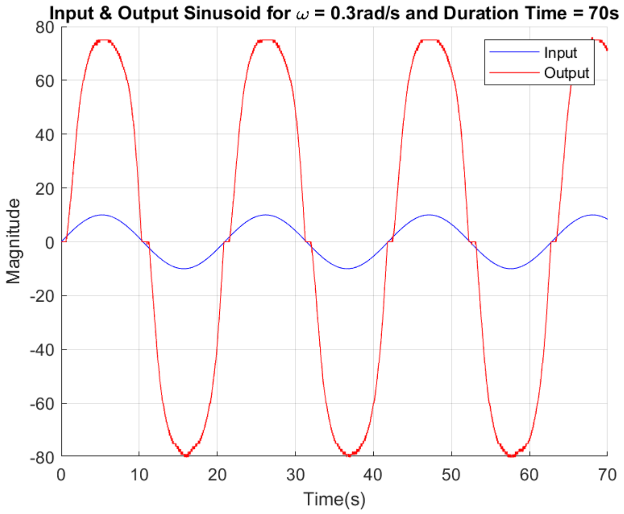

Fig. 3: Input and output signals (ω = 0.3 rad/s and t = 70s)

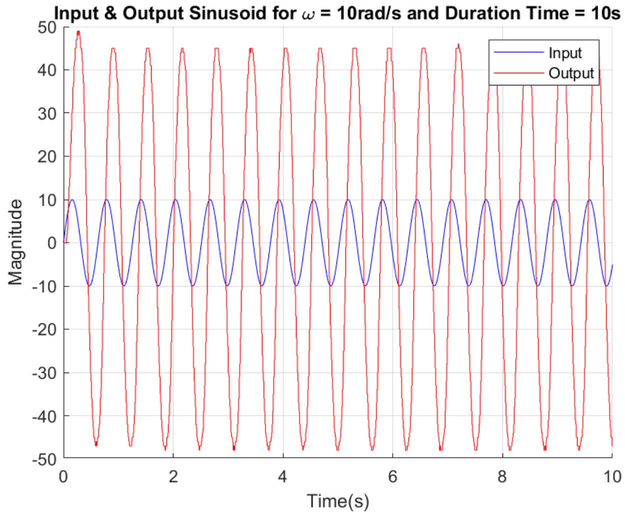

Fig. 6: Input and output signals (ω = 10 rad/s and t = 10s)

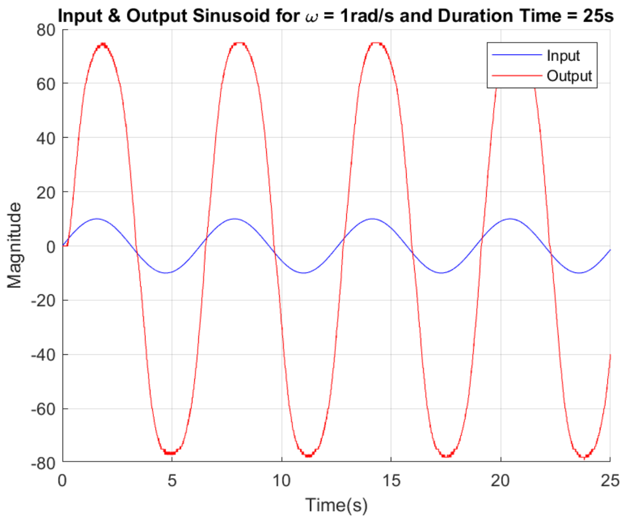

Fig. 4: Input and output signals (ω = 1 rad/s and t = 25s)

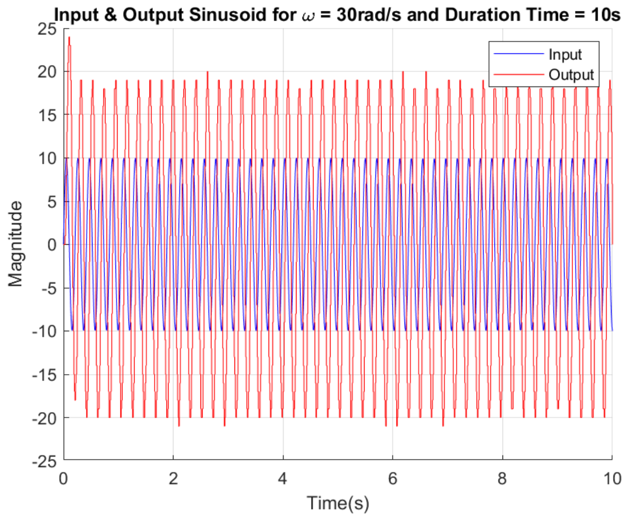

Fig. 7: Input and output signals (ω = 30 rad/s and t = 10s)

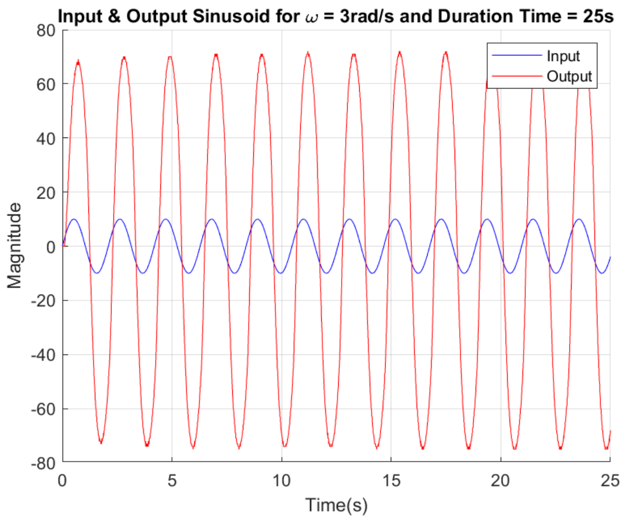

Fig. 5: Input and output signals (ω = 3 rad/s and t = 25s)

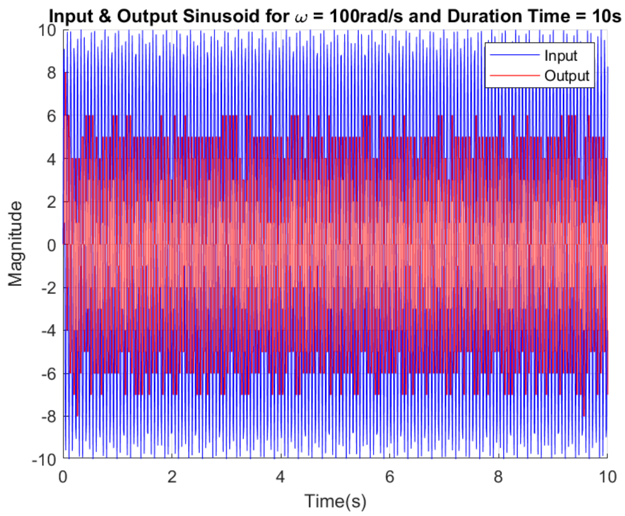

Fig. 8: Input and output signals (ω = 100 rad/s and t = 10s)

In each step, "fft" command of MATLAB is used in order to find the magnitude (dB) and phase (rad) values of that angular frequency and time duration. The list below shows the specific magnitude and phase values calculated for each angular velocity case.

Table 2: Calculated Magnitude (dB) and Phase (rad) values for each different angular velocity case

ω (rad/s)	Magnitude (dB)	Phase (rad)
0.1	18.2698 dB	-0.0619 rad
0.3	18.3126 dB	-0.0908 rad
1	18.2983 dB	-0.2570 rad
3	17.7326 dB	-0.6176 rad
10	13.6841 dB	-1.1576 rad
30	5.9321 dB	−1.6119 rad
100	-4.8298 dB	-2.4925 rad

Using these values, a comparison is made between the theoretical bode plot obtained in question 1, and simulation values whose values are calculated and listed above. Fig. 9 shows the Bode plot of magnitude (dB) and phase (rad).

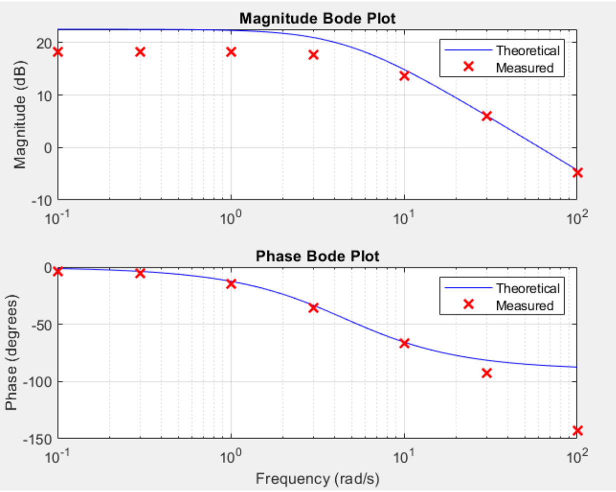

Fig. 9: Bode plot comparison between theoretical and simulation values

As seen from Fig. 9, simulation (measured) results are nearly on the theoretical curve. At low frequencies, measured results give a lower magnitude compared to the theoretical result. This could point to a potential energy loss in the system. Also, at high frequencies, measured phase values tend to be lower than the theoretical values, which is caused by the time delay of the actual system.

2.3 Q3: Pade's approximation and comparison

In this part, a first order Pade approximation is used to compensate the 10ms delay of the DC motor. The transfer function for first order Pade approximation is given below.

$$G_{delayed}(s) = \frac{1 - 0.005s}{1 + 0.005s}$$

Now, using the Pade approximation, the new Bode plot of the system is obtained. Fig. 10 shows all three results in the same plot, which are the theoretical value obtained in question 1 with no time delay, measured values obtained in question 2, and theoretical value with 10ms time delay obtained in question 3.

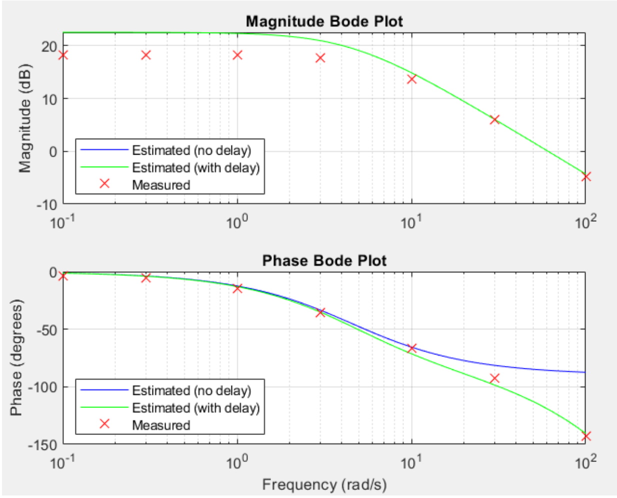

Fig. 10: Magnitude and phase Bode plot of the DC motor showing theoretical value with no time delay, theoretical value with 10ms time delay, and measured values on top

As seen from Fig. 10, at high frequencies, measured phase values are very close to the 10ms time delay transfer function graphs. Also, the magnitude of the theoretically calculated transfer functions are the same, just the phases differ due to presence of Pade approximation.

3. Conclusion

In this first part of the laboratory experiment, first order approximation of the estimated transfer function is implemented and the corresponding Bode plot of the DC motor is obtained. Then, in second part, a sinusoidal input is given into the system with 7 different angular frequency and time duration values. The corresponding output signals are plotted on the same plot with the input signal. Then, for each of seven different variables, magnitude and phase values are derived using the MATLAB's "fft" function. These derived values are compared with the Bode plot attained in the first question. At high frequencies, a mismatch is observed in phase plot due to the 10ms delay of the DC motor. Also, in magnitude plot, at low frequencies, the magnitude tend to be lower than the theoretical value. Lastly, in the last part of this laboratory experiment, Pade approximation is used in order to compensate the 10ms time delay of the DC motor. After obtaining the Bode plot relate with the Pade approximation, it is compared with the ones derived in previous parts. The overall result is, for the phase values, measured values in part 2 is very close to the values obtained from Pade approximation.

4. MATLAB CODE

PART 1

```
w = logspace(-1,2,100);
for k = 1:100
    s = 1i * w(k);
    G(k) = 13.42 / (1 + 0.22*s);
end
subplot(2,1,1)
semilogx(w,20*log10(abs(G)));
grid on
ylabel('Magnitude (dB)')
title('Magnitude Bode Plot')

subplot(2,1,2)
semilogx(w,angle(G)*180/pi)
grid on
xlabel('Frequency (rad/s)')
ylabel('Phase (degrees)')
title('Phase Bode Plot')
```

PART 2

```
ang_vel = 0.1; %CHANGE ACCORDINGLY
duration = 70; %CHANGE ACCORDINGLY
t = 0:0.01:duration;
input = 10*sin(ang_vel * t);

velocity = out.velocity;                 %
Simulink output (timeseries)

% ===== FFT-BASED MAGNITUDE (dB) AND PHASE
(rad) AT ω = 0.1 rad/s =====
Ts = 0.01;                               %
sampling time
Fs = 1/Ts;                                %
sampling frequency
y = velocity.data;
N = length(y);
x = input(1:N);

% FFT of input and output
X = fft(x);
Y = fft(y);

% Frequency vector in Hz
f = (0:N-1)*(Fs/N);

f_target = ang_vel/(2*pi);

[~, idx] = min(abs(f - f_target));

% Transfer function at that frequency
G(jω) = Y/X
G_mag = abs(Y(idx))/abs(X(idx));         %
MAGNITUDE
G_mag_dB = 20*log10(G_mag);              % dB
CONVERSION
G_phase = angle(Y(idx)) - angle(X(idx));
% phase in radians

% Store into your arrays
vals(7) = G_mag_dB;                       %
magnitude in dB
phases(7) = G_phase;                      %
phase in radians

fprintf('At ω = %.3f rad/s (f = %.4f
Hz):\n', ang_vel, f_target);
fprintf('Magnitude = %.4f dB\n',
G_mag_dB);
fprintf('Phase     = %.4f rad\n',
G_phase);

figure;
hold on;
plot(t, input, 'b');
plot(out.velocity, 'r');
title("Input & Output Sinusoid for \omega
= 0.1rad/s and Duration Time = 70s");
xlabel("Time(s)");
ylabel("Magnitude")
grid on
legend('Input', 'Output')

%% COMPARISON BETWEEN QUESTION 1 AND QUES-
TION 2
w = logspace(-1,2,100);

for k = 1:100
    s = 1i * w(k);
    G(k) = 13.42 / (1 + 0.22*s);
end

w_pts = [0.1 0.3 1 3 10 30 100];
mag_pts = [18.2698 18.3126 18.2983 17.7326
13.6841 5.9321 -4.8298];

phase_pts = unwrap([-0.0619 -0.0908 -
0.2570 -0.6176 -1.1576 -1.6119 -2.4925]);

% Convert phase from rad to degrees
phase_pts_deg = phase_pts * 180/pi;

figure;
subplot(2,1,1)
semilogx(w, 20*log10(abs(G)), 'b');
hold on
semilogx(w_pts, mag_pts, 'rx', 'Mark-
erSize', 8, 'LineWidth', 1.5);
grid on
ylabel('Magnitude (dB)')
title('Magnitude Bode Plot')
legend('Theoretical', 'Measured')

subplot(2,1,2)
semilogx(w, angle(G)*180/pi, 'b');
hold on
semilogx(w_pts, phase_pts_deg, 'rx',
'MarkerSize', 8, 'LineWidth', 1.5);
```

```
grid on
xlabel('Frequency (rad/s)')
ylabel('Phase (degrees)')
title('Phase Bode Plot')
legend('Theoretical', 'Measured')
```

PART 3

```
w = logspace(-1,2,100);
G_nom = zeros(size(w));   % estimated, no
delay
G_del = zeros(size(w));   % estimated,
with delay (Pade)

for k = 1:length(w)
    s = 1i * w(k);
    G_nom(k) = 13.42 / (1 + 0.22*s);
% original G(s)
    G_del(k) = G_nom(k) * (1 - 0.005*s) /
(1 + 0.005*s); % G_delayed(s)
end

w_pts     = [0.1 0.3 1 3 10 30 100];
mag_pts   = [18.2698 18.3126 18.2983
17.7326 13.6841 5.9321 -4.8298];
phase_pts = unwrap([-0.0619 -0.0908 -
0.2570 -0.6176 -1.1576 -1.6119 -2.4925]);
phase_pts_deg = phase_pts * 180/pi;

figure;
%magnitude
subplot(2,1,1)
semilogx(w, 20*log10(abs(G_nom)), 'b');
hold on
semilogx(w, 20*log10(abs(G_del)), 'g');
semilogx(w_pts, mag_pts, 'rx', 'Mark-
erSize',8);
grid on
ylabel('Magnitude (dB)')
title('Magnitude Bode Plot')
legend('Estimated (no delay)', 'Estimated
(with delay)', 'Measured', ...
       'Location','SouthWest')

%phase
subplot(2,1,2)
semilogx(w, angle(G_nom)*180/pi, 'b');
hold on
semilogx(w, angle(G_del)*180/pi, 'g');
semilogx(w_pts, phase_pts_deg, 'rx',
'MarkerSize',8);
grid on
xlabel('Frequency (rad/s)')
ylabel('Phase (degrees)')
title('Phase Bode Plot')
legend('Estimated (no delay)', 'Estimated
(with delay)', 'Measured', 'Loca-
tion','SouthWest')
```