EEE342 Lab #3 Report

Orkun İbrahim Kök

Department of Electrical and Electronics Engineering, Bilkent University, 06800 Ankara, Turkey

1. Introduction

The purpose of this laboratory experiment is to determine the gain, phase and delay margins of the system that is found in the first lab experiment. Moreover, the phase, gain, and delay margins of the system will be estimated by the mathematical model determined earlier, and will be verified analytically.

2. Laboratory Content

2.1 Margin Estimation

In the first part of the experiment, a controller plant is given, which contains a LPF and PI controller. The controller plant is given below.

$$G_c(s) = \left(\frac{1}{s + \tau_{LPF}}\right)\left(\frac{K_c(s+80)}{s}\right)$$

$$where\ K_c = \frac{2}{K_g}\ and\ \tau_{LPF} = \frac{3}{\tau_p}$$

Here, K_g is the DC gain and τ_p is the reciprocal of the distance of the pole of the first order approximation of the DC motor, that is found in LAB-1. The first order approximation found in LAB-1 is given below.

$$G_p = \frac{13.42}{0.22s + 1}$$

Calculation of K_g and τ_p values are given below.

$$K_c = \frac{2}{K_g} = 0.149$$

$$\tau_{LPF} = \frac{3}{\tau_p} = 13.636$$

Substituting these values into the controller plant yields to the following equation.

$$G_c(s) = \left(\frac{1}{s + 13.636}\right)\left(\frac{0.149(s+80)}{s}\right)$$

Finally, the first order Pade approximation is implemented to the system. The final version of the system is given below.

$$G(s) = G_p(s) \times G_c(s) \times \left(\frac{1 - 0.005s}{1 + 0.005s}\right)$$

The Bode plot of the system is plotted in MATLAB. Fig. 1 shows the Bode plot of the system.

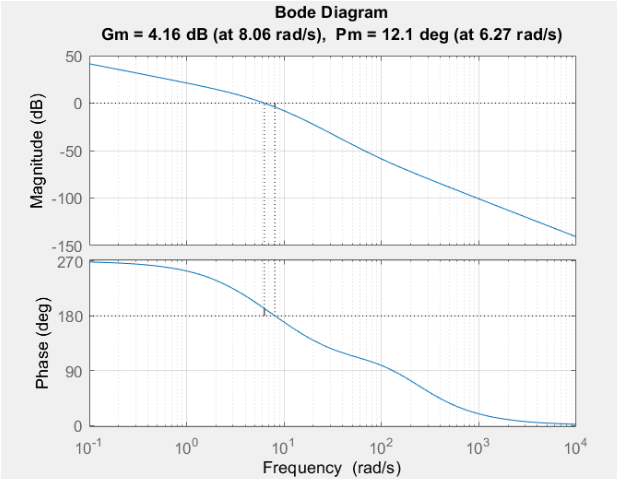

Fig. 1: Bode plot of the designed system.

As seen from Fig. 1, to determine the Gain margin, the gain of the system at 180 degrees, measured as 4.16dB. For the Phase margin, the phase at which the gain of the system is at 0dB is measured, which has a value of 12.1 degrees.

To calculate the Delay margin, the corresponding formula will be used.

$$DM = PM \times \frac{\pi}{180} \times \frac{1}{\omega_{gc}}$$
$$where\ \omega_{gc}\ is\ the\ PM\ frequency$$

$$DM = 12.1 \times \frac{\pi}{180} \times \frac{1}{6.27} = 0.0337\ seconds$$

This concludes the Margin Estimation part.

2.2 Margin Verification

In the second part of this laboratory experiment, the designed controller plant is implemented on the DC motor with an input signal of 40u(t) and the response is observed experimentally. After implementing the controller plant, gain and phase margin values were found by trial and error by looking at the plots generated in MATLAB. Firstly, the gain of the system in increased gradually and tried to find at which gain value out system becomes unstable, observing the MATLAB plots. The controller plant used in the system is given below, which is same as the previous step.

$$G_c(s) = 0.149\left(\frac{s + 80}{s^2 + 13.636s}\right)$$

Here, K (gain) value is 0.149 as calculated earlier.

Fig. 2 shows the velocity versus time plot of the system with a gain of K=0.08.

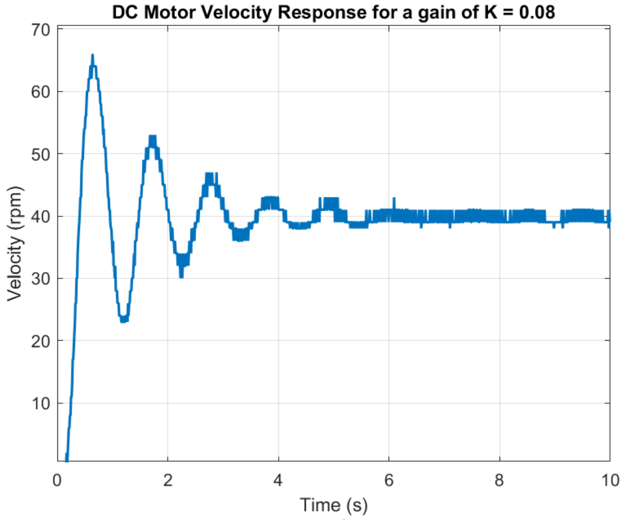

Fig. 3: Velocity vs Time plot of the system with gain K = 0.08.

As seen from Fig. 2, at K = 0.08 gain, the system is stable.

Fig. 3 shows the system with a gain of K = 0.18. As seen from the below figure, the system turned out to be marginally stable. The spikes in the voltage may be caused of the connections inside the DC motor.

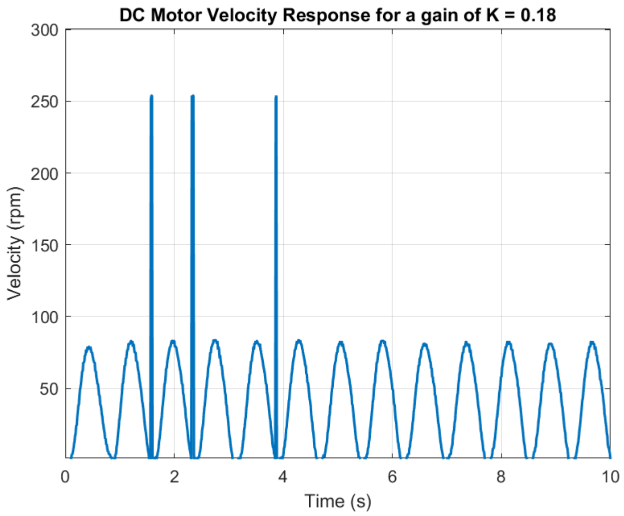

Fig. 4: Velocity vs Time plot of the system with gain K = 0.18.

Lastly, for the unstable case of the system, a gain of K = 0.25. As seen from Fig. 4, the system became unstable.

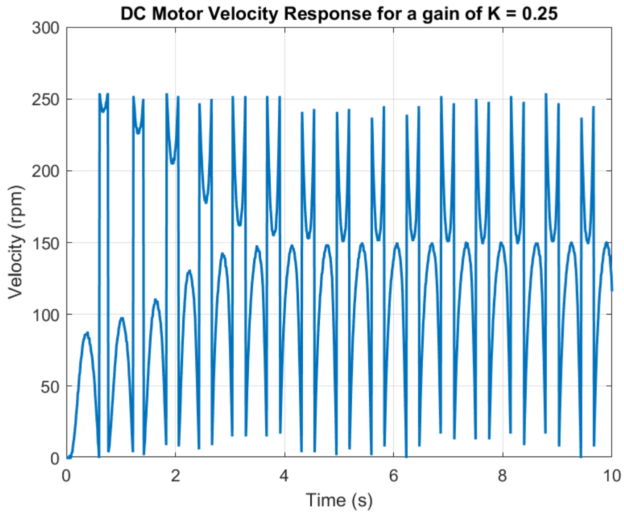

Fig. 2: Velocity vs Time plot of the system with gain K = 0.25.

Looking at Fig. 4, the critical gain value is found as K = 0.25. Using this value, GM of the system can be calculated using the below equations.

$$GM = \frac{K_{critical}}{K_{nominal}} = \frac{0.25}{0.149} = 1.678$$

$$GM_{db} = 20 \log GM = 4.496\ dB$$

In the previous part, GM of the designed system is measured as 4.16dB. The practical result is found as 4.496dB. The results are pretty close. Because we found the critical gain value by trial and error (increasing gain manually), a slight difference in the theoretical and practical may be present. This concludes the GM calculation of the DC motor with designed controller plant. Next task is determining the practical PM value of the system and comparing the found value to the theoretical value.

Fig. 5 shows the velocity versus time plot of the implemented controller plant system with a time delay of 0.001 seconds.

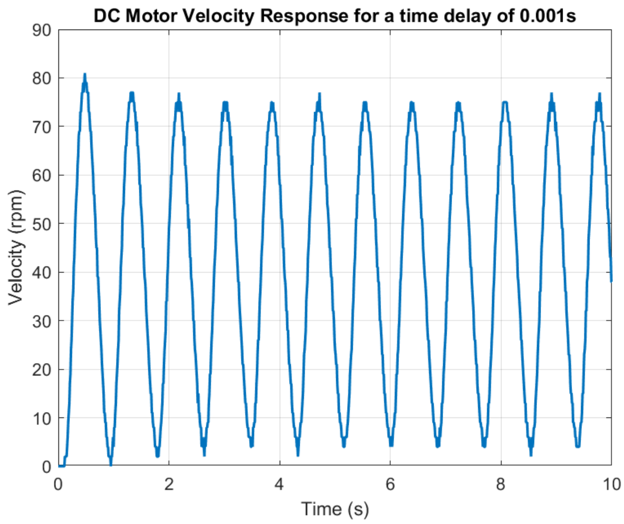

Fig. 5: Velocity vs Time plot of the system with time delay of 0.001s.

As seen from Fig. 5, at a time delay of 0.001s, system is stable.

Fig. 6 shows the corresponding velocity versus time plot for a time delay of 0.003s. The corresponding shows that with that time delay value, system is marginally stable.

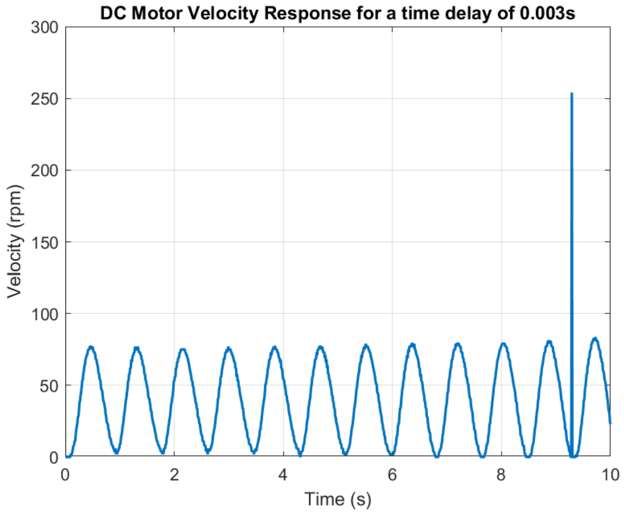

Fig. 6: Velocity vs Time plot of the system with time delay of 0.003s.

Furthermore, the voltage spike may be caused by the hardware of the DC motor.

Lastly, to find where the system became unstable, a time delay value of 0.03s.

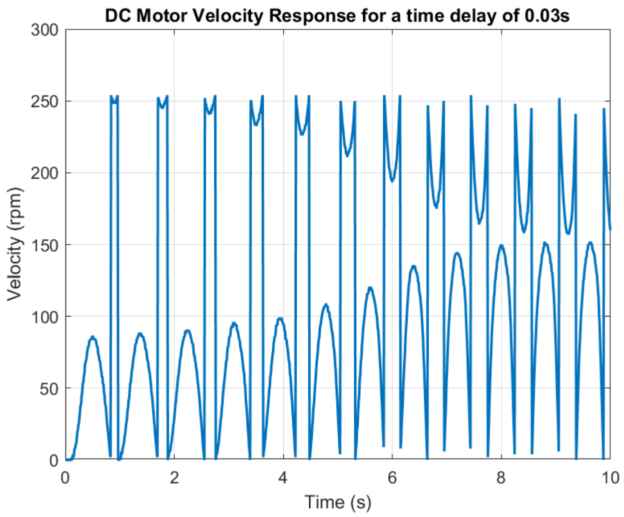

Fig. 7: Velocity vs Time plot of the system with time delay of 0.03s.

As seen from Fig. 7, the system became unstable due to the oscillations present in the plot. To calculate the PM of the system, below equation will be used.

$$PM = DM \times \omega_{gc} \times \frac{180}{\pi}$$

$$where\ \omega_{gc}\ is\ the\ PM\ frequency$$

$$PM = 0.03 \times 6.27 \times \frac{180}{\pi} = 10.777°$$

The found PM value is 10.777 degrees, whereas, in part 1, PM value is measured as 12.1 degrees. The obtained theoretical and practical results are quiet close.

Additionally, in part 1, the DM of the system is calculated as 0.0337 seconds, whereas in the practical measurements, DM value is measured as 0.03 seconds by trial and error. Again both PM and DM values are close to values comparing the theoretical and practical values.

3. Conclusion

Throughout this laboratory experiment, margins of a designed system are estimated, and then verified using a real DC motor in real life. In the first part, a controller plant is implemented using the obtained first order approximation in the first lab experiment. Adding a Pade approximation to the plant, we attained out system for the DC motor. Using it in the MATLAB, we obtained the margin values. Then, we moved on to the practical experimentation. By trial and error, we tried to detect where the system becomes unstable changing the delay or gain variables. At the end, we obtained values for both delay and gain variables by trial and error looking at the velocity versus time plots. Comparing the theoretical and practical values concluded this laboratory experiment. Overall, the compared results turned out to be quiet close, which shows that this laboratory experiment is accomplished.

4. MATLAB Code

Part 1

```
s = tf('s');

% Plant
Gp = 13.42/(0.22*s + 1);

% Controller parameters
Kg = 13.42;
tau_p = 0.22;

Kc = 2/Kg;              % = 0.149
tau_LPF = 3/tau_p;      % = 13.636

% Controller
Gc = (1/(s + tau_LPF)) * (Kc*(s + 80)/s);

% First-order Pade approx for 0.005 s
Delay = (1 - 0.005*s)/(1 + 0.005*s);

% Open-loop transfer function
G = Gc * Gp * Delay;

figure;
margin(G)
grid on
```

Part 2

```
t = velocity.Time;
y = squeeze(velocity.Data);             %
Converts 1x1x1001 → 1001x1
delay = 0.02;
```

```
plot(t, y, 'LineWidth', 1.5);
grid on;
xlabel('Time (s)');
ylabel('Velocity (rpm)');
title('DC Motor Velocity Response for a
time delay of',delay);
```